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Width-dependent characteristics of pile/soil system subjected to lateral force.

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Width - Dependent Characteristics of Pile/Soil System Subjected to Lateral Force

By

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Abstract

This thesis deals with the determination of modulus of subgrade reaction for both sandy and clayey soils surrounding laterally loaded short piles. The initial value of modulus of subgrade reaction determined is based on Terzaghi, Bowles' and Vesic's method.

The difference in lateral displacement of models from laboratory tests and that from numerical analysis are associated with inaccurately determined values of modulus of subgrade reaction. The evaluation of modulus of subgrade reaction, which presents the same value of horizontal displacement at the top of pile determined in laboratory experiment and obtained from numerical investigation, is performed in the scope of sensitivity analysis.

To my family

ACKNOWLEDGEMENTS

I am pleased to express my deep and sincere thanks to my advisor, ***Dr. B.Budkowska*** for her continuous support, encouragement and valuable recommendations without which this work would have not been accomplished.

I also wish to thank the Civil Engineering faculty members and staff.

Finally, I would like to thank my mother, wife and my young son (***Mohab***) for asking God for my success.

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Notation

w	Water content of the soil
W_w	Weight of water present in soil mass
W_s	Weight of soil solids
W_t	Total weight of the soil
V_w	Volume of water present in the soil mass
V_v	Volume of soil void
V_S	Volume of soil solids
γ_d	Dry unit weight of soil
c	Cohesion
q_u	Unconfined compressive strength
ε	Unit strain of the soil
ΔL	Total sample deformation (axial), for unconfined compression test
L_O	Original sample length, for unconfined compression test
σ	Normal stress of the soil
P	Acting load on the sample at any instant for corresponding value ΔL
A'	Actual cross sectional area of specimen for the corresponding load P
A_O	Original area of specimen
d	Diameter of specimen
E_S	Modulus of elasticity of the soil
B	Width of the pile's cross-section
h	Height of the pile cross-section
H	Length of the pile
H_1	Pile length above the soil level
H_2	Pile length from the soil level to the point of rotation
H_3	Pile length between the point of rotation and the bottom of the pile

F	Lateral force applied at the top of the pile
z	Currant spatial variable along the pile axis
v	Lateral displacement at any arbitrary point for primary structure
u	Vertical displacement at any arbitrary point for primary structure
v''	Second derivative of lateral displacement at any arbitrary point for primary structure
v_{Top}	Lateral displacement at the top of the pile for primary structure
v_{bot}	Lateral displacement at the bottom of the pile for primary structure
$v_{T(B)}$	Lateral displacement at the top of the pile for primary structure using BELF
$v_{D(B)}$	Lateral displacement at the bottom of the pile for primary structure using BELF
v^1	Lateral displacement at the top of the pile for adjoint structure
$\overline{v}$	Lateral displacement at any arbitrary point for adjoint structure
$\overline{v}''$	Second derivative of lateral displacement at any arbitrary point for adjoint structure
δv	First variation of lateral displacement imposed on the primary structure
$\delta v''$	First variation of second derivative of lateral displacement imposed on the primary structure
p	Soil reaction of primary structure
$\overline{p}$	Internal soil reaction of the adjoint structure
p_{top}	Soil reaction at the top of the pile
p_{bot}	Soil reaction at the bottom of the pile
M	Internal bending moment of primary structure
$\overline{M}$	Internal bending moment of the adjoint structure
k_h	Horizontal modulus of subgrade reaction
$k_h(B)$	Horizontal modulus of subgrade reaction associated with width B of the pile
k_s	Vertical modulus of subgrade reaction
δk_h	Correction (inaccuracy/first variation) of modulus of subgrade reaction

n_h Constant of horizontal modulus of subgrade reaction

$n_h(B)$ Constant of horizontal modulus of subgrade reaction associated with width B of the pile

δn_h Correction (inaccuracy/first variation) of horizontal modulus of subgrade reaction

E_P Modulus of elasticity of the pile

I_P Moment of inertia of the pile

μ_S Poisson's ratio

λ Characteristic pile's length

Chapter 1

Introduction

Early designers assumed piles could carry only an axial load with graphical methods being used to find the individual pile loads in group. In this case a force polygon containing horizontal forces required battered piles, to carry the horizontal load as a component of the axial load. Sign posts, power poles, and many marine pilings represented a large class of partially embedded piles subject to lateral loads which tended to be designed as laterally loaded piles. Current practice treats the full range of slender vertical structural members, fully or partially embedded in the ground, as laterally loaded piles. A large number of load tests have fully validated concept of vertical piles being capable of carrying lateral loads via shear and bending rather than as axially loaded members.

The piles subjected to lateral loading are usually analyzed as the beam on elastic foundation of Winkler type. This model enjoys a great popularity among engineers because of its simplicity, it is also simple to numerical analysis by finite element method. The major difficulty the model encounters in practice is connected with determination of reliable value of modulus of subgrade reaction k appropriate for analysis of piles.

Based on elastic continuum theory Terzaghi defined modulus of subgrade reaction k as the quotient of the pressure applied to the surface and the produced displacement. The value of modulus of subgrade reaction depends on elastic properties of the subgrade and the dimensions of the area acted upon by the subgrade reaction

This approximation leads directly to width dependent modulus of subgrade reaction. However, the relationship proposed by Terzaghi when substituted in the differential equation of beam on Winkler foundation, results in constant value of soil response for any pile with arbitrary width. It is worth reminding that analysis of beam on elastic foundation is based on plane stress approached. Consequently k is treated in similar way to uniformly distributed load acting on a beam, it can always be replaced by line load and beam is reduced to a linear element. On the other hand, the modulus of subgrade reaction determined by means of elastic continuum theory is based on plane strain formulation.

The adoption of concept of beam on elastic foundation to analysis of laterally loaded piles can lead to inaccurate results. It is caused by the fact that the pile is not rotated by 90° beam on Winkler foundation but it is a column embedded and surrounded by soil medium and subjected to lateral forces. It is easy noting that soil response of laterally loaded pile is more complex than Winkler foundation model can offer. Simplification of that complex behavior to the beam on elastic foundation reflects on different relationship for k than that which is used in beam theory. Also, the laterally loaded piles are usually designed in accordance with serviceability requirements where the maximum lateral displacement is of concern. There exists a substantial amount of technical literature on the analysis of deep-foundation subject to lateral loading.

The model of a beam on elastic foundation (linear and nonlinear) is mostly used in engineering practice. The $p-y$ method Reese (1977) developed in seventies for different types of soils enjoy high popularity. It was developed on full-scale models. The quintessence of $p-y$ method is embodied in the fact that it allows for determination of nonlinear relationship between soil reaction p and the lateral displacement at discrete points located along the pile axis. Among various soil parameters required for development of $p-y$ model, there is also the modulus of subgrade reaction k, which is used in analysis of beams on elastic foundation of Winkler type. Terzaghi was the first who proposed the simple method for determination of k. Although he did not present any experimental evidence for k values for various types of soils that he presented, his method of determination of k suffers some shortcomings. The most serious is negligence of shear effect affecting the equation of static equilibrium. The important aspect, which is worth noting, is connected with the fact that in analysis of laterally loaded piles the upper part of soil which depth is equal to maximum length of short pile affects the behavior of the pile.

It is worth adding that in contrast to Terzaghi's static method, the $p-y$ method requires installation of expensive instrumentation located at various points along the pile's length. The development of sensitivity theory and its application to material identification allows overcoming the described above shortcomings.

The objective of this research is to develop a method of identification of modulus of subgrade reaction for pile laterally loaded and embedded in homogenous sandy and clayey soil. This method enables one to evaluate the correct value of modulus of subgrade reaction based on its initial value determined in laboratory. Also to enables one to know if modulus of subgrade reaction is dependent of the pile's width or not.

Literature Review is presented in Chapter 2 of these research, making review of research on various aspects of laterally loaded pile and determination of modulus of subgrade reaction.

In Chapter 3, material properties such as water content, liquid limit, plastic limit and unconfined compressive strength for different composition of clayey soil consisting of different amount of sand, nontreated bentonite and lime were determined to obtain all information about investigated soil and to use it for initial modulus of subgrade reaction determination which is described in Chapter 5.

In Chapter 4, the laboratory tests are presented for many cross-sections of pile samples with different widths and constant heights and lengths .The test was conducted by driving the model of the pile into both sandy and clay soil and laterally loaded them by lateral forces at the top of the model which was increased from zero to its final value in incremental fashion (failure of model of the pile is observed, when the small increment load caused large deflection at the top point of the model of the pile).

Also values of modulus of subgrade reaction are determined as a first approximation for sample model piles embedded in sandy and clay soil in Chapter 4 by Terzaghi's method. It is postulated that short rigid pile in elastic range subjected to lateral load deforms through the rigid rotation. Noting that the variability of modulus of subgrade reaction along the pile penetrating clay soil has constant value, while for sand soil, it varies in linear fashion.

Determination of initial modulus of subgrade reaction was also determined by both Vesic's and Bowles' method. Based on modulus of elasticity of soil, dimensions (width of the pile) and properties for each cross section (modulus of elasticity of the pile), the modulus of subgrade reaction was obtained by Vesic's method. Based on unconfined shear strength of soil, which was presented in Chapter 3, the modulus of subgrade reaction was also determined by Bowles'method. Some of these formulas take into account width as well as pile's stiffness (Vesic's method) while other are independent of width (Bowles' method). The approximated assessment of k by means of various formulas often results in some irregularities.

All values of initial modulus of subgrade reaction, which calculated formerly in Chapter 5 were used in analysis of the models of the piles structures subjected to lateral load by means of numerical analysis employing the Finite Element Method (BELF), in Chapter 6. By discretizing the model of the pile into small elements and evaluating the lateral displacement at the top node due to a certain lateral force. The numerical results in terms of the lateral displacements were compared with those determined in laboratory (Chapter 4). Considerable difference between numerical and experimental results was observed.

Due to the inaccuracy of the results obtained in both experimental tests and numerical analysis, in Chapter 7, the theory of sensitivity analysis of laterally loaded piles is presented .By knowing displacement at the top of the model of the pile in both laboratory and that obtained by numerical analysis, the first variation of the displacement was determined. Employing this variation leads to calculation of the first variation of modulus of subgrade reaction, which added to the initial value of modulus of subgrade reaction producing the corrected value, which reused again by numerical analysis to evaluate the corrected lateral displacement very close to the laboratory results.

At the end of the research, the analysis of experimental results and discussion on identification investigation are conducted, showing the parameter associated with soil behavior surrounding laterally loaded piles.

Chapter 2

Literature Review

The review of research on various aspects of laterally loaded pile and determination of modulus of subgrade reaction is presented below.

The concept of subgrade reaction was introduced into applied mechanics by Winkler (1867), and was used by Zimmermann (1888) for the purpose of computing the stresses in railroad ties, which rest on ballast over their full length. During the following decades the theory was expanded to include the computation of stresses in flexible foundation, such as continuous footings or rafts and in concrete pavements acted upon by wheel loads.

Investigators who are primarily interested in the theoretical aspects of the problem have written most of the papers dealing with problems of subgrade reaction. They published the solution of the differential equation, taking it for granted that the value of the coefficient of subgrade reaction be known. Hayashi (1921), in his comprehensive treatise on the subject, notified that the vertical modulus of subgrade reaction should be determined by loading test, but he did not mention the fact that the results of a loading test depend on the size of the loaded area. The book by Hetenyi (1946) on beams on elastic foundations does not contain any statement regarding the factors, which determine the numerical value of coefficient of subgrade reaction.

Terzaghi (1932) published a paper, which dealt with the factors used for determination of the value of vertical modulus of subgrade reaction for flexible raft foundation, acted upon by line loads. The Terzaghi's next paper (1955) that considered factors which used for evaluation of the numerical value of the coefficients of vertical and horizontal subgrade reaction for sand and clay under simple and frequently encountered conditions. It also contained a description of the procedures by means of which reasonable values for the coefficients of vertical and horizontal subgrade reaction can be secured.

Budhu and Davies (1987) presented the results of a numerical analysis of single laterally loaded piles embedded in cohesionless soils by taking soil yielding into account. The input parameters for the soil were the angle of internal friction and a parameter characterizing the increase in soil stiffness with depth and they were assumed to be linear. The analysis showed that soil yielding greatly increases the displacements, rotations, and bending moments of laterally loaded piles.

Meyerhoff and Ghosh (1989) determined under various combinations of eccentricity and inclination of the load varying in direction from vertical to horizontal the ultimate bearing capacity of flexible single model piles as well as small pile groups of timber and nylon in loose sand and soft clay. They presented the results of the load tests in the form of polar bearing capacity diagrams and compared them with theoretical estimates based on the concept of an effective embedment depth in terms of the behaviour of equivalent rigid piles. The results of model tests on single flexible piles under eccentric inclined loads in loose sand and clay showed that the eccentricity and inclination of the load significantly influenced the ultimate bearing capacity of flexible piles. The vertical component of ultimate eccentric inclined load for flexible piles and pile groups in sand can be approximately obtained by multiplying the ultimate axial bearing capacity of piles and pile groups with an eccentric inclination factor.

Joshi, Sharma and Sparrow (1989) presented research on the instrumented model piles, which were loaded to failure using slow maintained load (Slow-ML), quick maintained load, and constant-rate-of-penetration (CRP) methods of loading. The piles were driven in a prepared dry sand bed. The applied load, point load, and shaft resistance were measured using load cells and strain gauges, and axial force distribution was determined. Conclusions made were:

1. If the pile load test is performed to determine the ultimate pile capacity, it does not matter which test method is selected. If time is a constraint, then CRP test method is recommended.

2. When the load versus displacement curve is desired for conditions occurring during the construction process, the Slow-ML test method is the most realistic procedure.

3. Axial force distribution and shaft resistance distribution along the pile are identical regardless of pile test method.

Behaviour of flexible piles under inclined loads was considered by Sastry and Meyerhoff (1990). Research was done on the lateral soil pressures, bending moments, pile displacements at ground surface, and bearing capacity of instrumented vertical single flexible model piles in homogeneous loose sand and soft clay under central inclined loads. The axial piles capacity of a flexible pile will be unchanged from that of a rigid pile, whereas under lateral load, the capacity of a flexible pile can be estimated using the concept of an " effective embedment depth of an equivalent rigid pile.

The bearing capacity of flexible model piles and small pile groups under axial, lateral, and various combinations of eccentric and inclined loads in layered soil consisting of clay overlying sand was investigated by Yalcin and Meyerhoff (1991). It was found that in absence of structural pile failure the ultimate loads depend not only on the eccentricity and inclination of the load but also on the ratio of the upper clay soil layer thickness to pile embedment. The results showed that the ultimate bearing capacity varies with load eccentricity and inclination and with the ratio of upper soil thickness to pile embedment in a similar way to that of single piles.

Disagreeing with classical Winkler's model to represent an elastic continuum, Vlasov and Leontev (1990) developed a two-parameter model for a beam on an elastic foundation. In their paper Vallabhan and Das (1991) reached the following conclusions:

1. It is shown that the value of the modulus of subgrade reaction depends on the depth of the soil, distribution of loading, and stiffness of the beam and the soil.

2. The writers have verified that the loading is either uniform or even linearly varying on the entire beam.

3. The elastic material properties of the soil are employed.

4. The Vlasov model can be very easily modified to consider variation of material properties with depth.

A paper on lateral pile response to monotonic pile head loading was published by Yan and Byme (1992). This paper presented results from a series of model tests of single vertical piles subjected to lateral monotonic pile head loading. Model tests were carried out in sand under simulated field stress conditions using the hydraulic gradient similitude technique. The focus of studies was on examining various factors that affect the soil-pile interaction in terms of p-y curves. The tests showed that experimental p-y curves are nonlinear and stress dependent. Comparison was made between experimental p-y curves and those recommended by the American Petroleum Institute (1990). The proposed parabolic p-y curves can better resemble experimental p-y curves and give a better prediction of pile response for both free and fixed head conditions.

The main objective of the work presented in the paper by Anagnostopoulos and Georgiadis (1993) was to investigate experimentally any possible effects of lateral loading on axial pile displacements and stresses as well as the influence of axial loads on the lateral pile response. They concluded:

1. The lateral load increased the axial pile displacement. The amount of increase was dependent on the value of the lateral load and on the level of the axial load.

2. The lateral load caused a small reduction of the axial pile stresses near the ground surface and appeared to have rather limited effects on the ultimate axial load.

3. The effect of axial loading on the lateral pile response was rather limited.

4. The interaction between axial and lateral pile response can be studied with a nonlinear finite-element analysis.

Investigation was done by Sastry and Meyerhoff (1994) on the lateral soil pressures, bending moments, pile displacements at ground surface, and bearing capacity of instrumented vertical single flexible model pile in layered sands consisting of loose sand overlying compacted sand under vertical eccentric and central loads. The results of these load tests were compared with theoretical estimates based on the concept of an effective embedment depth of equivalent rigid piles.

Behaviour of flexible piles in layered clays under eccentric and inclined loads was studied by Sastry and Meyerhoff (1995). Investigation was done on the lateral soil pressure, bending moments, pile displacements at the ground surface, and the bearing capacity of instrumented vertical single flexible model piles in layered clay system consisting of medium clay overlying soft clay under eccentric and central inclined loads.

The lateral soil pressures on the upper portion of the pile shaft of a flexible pile under pure moment and lateral load can be estimated from those on an equivalent rigid pile.

Previous analysis of the ultimate resistance and displacements of flexible piles under lateral loads in cohesionless soils was re-evaluated by Meyerhoff (1995) by using correlations based on standard penetration tests. He concluded that for cohesionless soils preliminary estimates of the behaviour of piles under lateral loads could, as in case of vertical loads, also be made from the results of standard penetration tests.

A method to predict the load-displacement relationship for single piles subjected to lateral load, embedded in sand, considering soil nonlinearity using subgrade reaction model was developed by Prakash and Kumai (1996). The following conclusions were drawn:

1. For a given relative density of sand, the modulus of horizontal subgrade reaction is an exponential function of strain.
2. Modulus of horizontal subgrade reaction depends on the relative density of sands and the position of the ground-water table.
3. A range of horizontal modulus of subgrade reaction values for sands have been proposed for different relative densities for use in practice. Pile type and pile diameter have little effect on horizontal modulus of subgrade reaction values.
4. A design procedure has been proposed to predict the load-deflection curve considering soil nonlinearity.
5. For piles in dense sands, the maximum variation in predicting load using the p-y method is approximately two times bigger that of the proposed method. For medium sands, both methods predict reasonably close loads, but the proposed method is more rational and easy to use.

6. The maximum variation in the load-deflection plot using this method for loose sands is about 80%. Therefore, more data for piles in loose sands need to be analyzed to narrow the range of horizontal modulus of subgrade reaction values.

In their study Prasad and Rao (1996) investigated the behaviour of piles under lateral loads in clayey soils. They concluded that the lateral capacity of helical piles increases with embedded depth and shear strength of the soil. The capacity of helical piles is greater than the capacity of a single pile shaft and increases with the number of helical plates. The capacity of helical piles is 1.2-1.5 times the capacity of a single straight pile shaft without plates.

Chapter 3

Engineering Properties of Soils and Their Measurements

It is important to know some properties of the soil, which will be tested to help one for knowing the behavior and characteristics of the soil, as water content, liquid and plastic limit, grain size distribution of the soil and compressive strength.

The following describes the types of soils used in experimental tests.

1. Sandy Soil

The sand used for the model pile tests was well graded and had medium to coarse angular grains.

2. Clayey Soil

The clay used for the model pile tests had a medium plasticity and was made brittle by the addition a suitable amount of lime (by adding deferent amounts for each type to obtain more brittle material). It consists of sand, nontreated bentonite clay and lime and it is divided into three types as follows:

Soil Type 1

It is a mix of 50% sand, 50% nontreated bentonite clay.

Soil Type 2

It is a mix of 40% sand, 60% nontreated bentonite clay.

Soil Type 3

It is a mix of 25% sand, 75% nontreated bentonite clay.

3.1 Water Content Determination

3.1.1 Objective

To obtain the water content of the sample.

3.1.2 Determination of Water Content for the Investigated Soils

The standard procedures were used for evaluation the water contents for the different soils.

1. Sandy Soil

- Evaluation of the Water Content of Sandy Soil with 1.5% Lime

- **The average water content = 2.48% (dry sand)**

2. Clayey Soil

The water content was evaluated for all types of clayey soil that described previously in this chapter with different amount of lime added to make the sample brittle material.

2.1 Soil Type 1

Different amounts of lime were added to the mix of 50% sand and 50% nontreated bentonite.

2.1.1 Soil Type 1(with 1.5 % Lime)

- Evaluation of the Water Content of Sample Type 1 with 1.5% Lime
- **water content = 40.7%**

2.1.2 Soil Type 1(with 2.0 % Lime)

- Evaluation of the Water Content of Sample Type 1 with 2.0% Lime

- **water content = 41.6% (dry sand)**

2.1.3 Soil Type 1(with 2.5 % Lime)

- Evaluation of the Water Content of Sample Type 1 with 2.5% Lime

- **water content = 41.02%**

Water content = **41.6%** with 2% Lime was more brittle after drying for soil type 1

2.2 Soil Type 2

Different amounts of lime were added to the mix of 40% sand and 60% nontreated bentonite.

2.2.1 Soil Type 2(with 1.5 % Lime)

- Evaluation of the Water Content of Sample Type 2 with 1.5% Lime
- **water content = 40.1%**

2.2.2 Soil Type 2(with 2.0 % Lime)

- Evaluation of the Water Content of Sample Type 2 with 2.0% Lime

- **water content = 39.0%**

2.2.3 Soil Type 2(with 2.0 % lime)

- Evaluation of the Water Content of Sample Type 2 with 2.5% Lime

- **water content = 42.3%**

Water content = **39.0%** with 2% lime was more brittle after drying for soil type 2

2.3 Soil Type 3

Different amounts of lime were added to the mix of 25% sand and 75% nontreated bentonite.

2.3.1 Soil Type 3(with 1.5 % Lime)

- Evaluation of the Water Content of Sample Type 3 with 1.5% Lime
- **water content = 42.45%**

2.3.2 Soil Type 3(with 2.0 % Lime)

- Evaluation of the Water Content of Sample Type 3 with 2.0% Lime
- **water content = 40.3%**

2.3.3 Soil Type 3(with 2.5 % lime)

- Evaluation of the Water Content of Sample Type 3 with 2.5% Lime
- **water content = 41.17%**

Water content = **40.3%** with 2% lime was more brittle after drying for soil type 3

3.2 Liquid Limit of a Soil

3.2.1 Objective of the Experimental

To determine the liquid of investigated soil.

3.2.2 Determination of Liquid Limit for the Investigated Soils

The standard procedures were used for evaluation the liquid limit for the different soils.

1. Soil Type 1

- Evaluation of Liquid Limit of Sample Type 1
- **Liquid Limit = 47 %**

2. Soil Type 2.

- Evaluation of Liquid Limit of Sample Type 2
- **Liquid Limit = 49 %**

3. Soil Type 3.

- Evaluation of Liquid Limit of Sample Type 3
- **Liquid Limit = 51 %**

3.3 Determination of Plastic Limit for the Investigated Soils

1. Soil Type 1

- Evaluation of Plastic Limit of Sample Type 1
- **Plastic Limit. = 15.9 %**

2. Soil Type 2

- Evaluation of Plastic Limit of Sample Type 2
- **Plastic Limit. = 17.0 %**

3. Soil Type 3

- Evaluation of Plastic Limit of Sample Type 3
- **Plastic Limit. = 17.4 %**

3.4 Grain-Size Analysis (Mechanical Method)

3.4.1 Objective of the Experimental

To obtain the method of making a mechanical grain-size analysis of a soil and presenting the resulting data

3.4.2 Determination of Grain Size Distribution for the Investigated Soil

1. Soil Type 1

$$\%Losses = (1 - \frac{497.6}{500}) * 100 = 0.48\ \%$$

2. Soil Type 2

$$\%Losses = (1 - \frac{498.5}{500}) * 100 = 0.30\ \%$$

3. Soil Type 3

$$\%Losses = (1 - \frac{498.1}{500}) * 100 = 0.38\ \%$$

3.5 Unconfined Compression Test

3.5.1 Objective

To evaluate the shear strength of a cohesive soil and using it for determination the modulus of subgrade reaction by Bowles'method. Also to determine the modulus of elasticity and using it for determination the modulus of subgrade reaction by Vesic's method.

3.5.2 Determination of the Unconfined Compressive Strength for the Investigated Soils

1. Soil Type 1

- Evaluation the Unconfined Compressive Strength of Sample Type 1

$q_u = 0.5\ ksf = 23.94\ kPa$

2. Soil Type 2

- Evaluation the Unconfined Compressive Strength of Sample Type 2

$q_u = 0.55\ ksf = 26.33\ kPa$

3. Soil Type 3

- Evaluation the Unconfined Compressive Strength of Sample Type 3

$q_u = 0.47\ ksf = 22.5\ kPa$

3. 6 Determination of Modulus of Elasticity for the Soils

1. Soil Type 1

$$E_s = \frac{0.146}{0.0057} = 25.6\ ksf = 1226\ kPa$$

2. Soil Type 2

$$E_s = \frac{0.177}{0.0057} = 31.05\ ksf = 1486\ kPa$$

3. Soil Type 3

$$E_s = \frac{0.1428}{0.005} = 28.56\ ksf = 1367\ kPa$$

Chapter 4

Experimental Procedure and Laboratory Tests

4.1 Purpose of Experiment

The purpose of the experiment was to obtain the lateral displacement at the upper end of the model of the pile due to later forces applied to the same point (upper end of the pile).

Two cases were used for test:

1. Models of pile structure having variable widths ($B_1, B_2 \cdots B_8$) and the same length embedded in clayey soil and subjected to lateral forces.
2. Models of pile structure having variable widths ($B_1, B_2 \cdots B_8$) and the same length embedded in sandy soil and subjected to lateral forces.

4.2 Cross Section of the Samples of the Models of the Piles

Many models of the piles were used with different widths B_i in one direction and keeping the length of the piles constant. Also the height in other direction remained constant to make only one variable (B_i) as shown in Fig.4.1

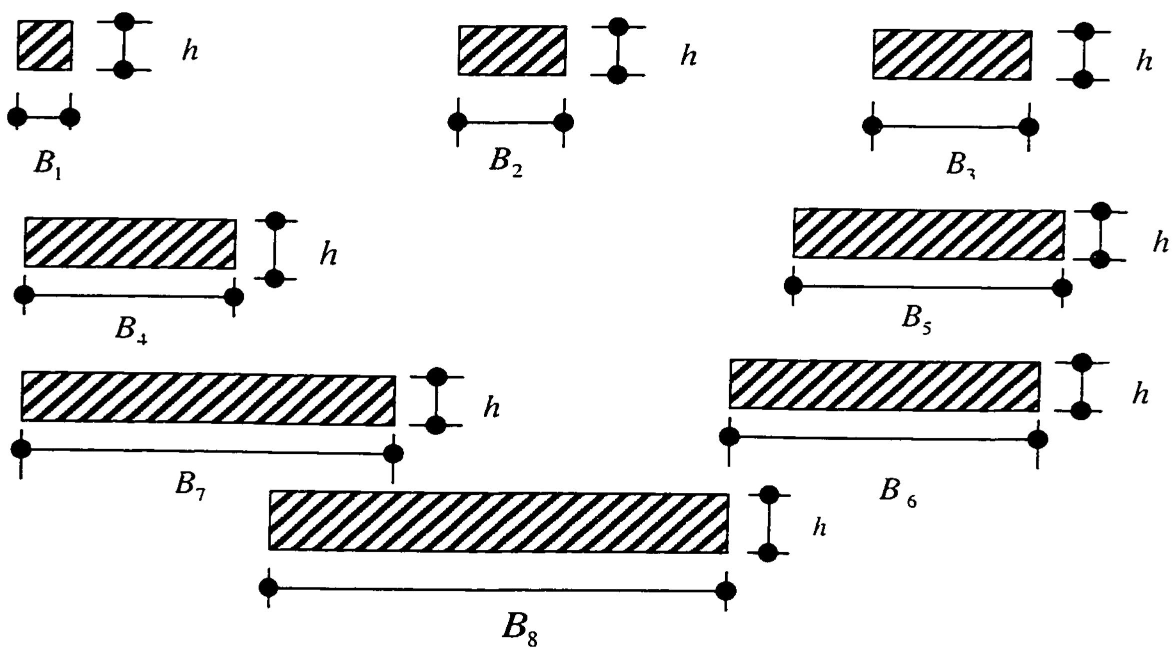

Figure 4.1. The Different Cross Section of the Models of the Piles.

where the dimensions of the models of the piles cross sections are as follows:

$h = 6.35\ mm$

$B_1 = 6.35\ mm$ $B_2 = 12.7\ mm$

$B_3 = 19.05\ mm$ $B_4 = 25.4\ mm$

$B_5 = 31.75\ mm$ $B_6 = 38.1\ mm$

$B_7 = 44.45\ mm$ $B_8 = 50.8\ mm$

Length of the pile:

$L = 400\ mm$ (For clayey soil) - $L = 350\ mm$ (for sandy soil)

4.3 Preparation of the Testing Container

The testing container shown in Fig.4.2 was 0.8 m wide, 2 m long and 1.7 m high. It is made of 15 mm thick Plexiglas. In order to test the model of the pile structures in clayey soil, the testing container was filled with clay in 200 mm layers. Each layer was compacted using a wooden impact hammer to have consistently compacted clayey soil.

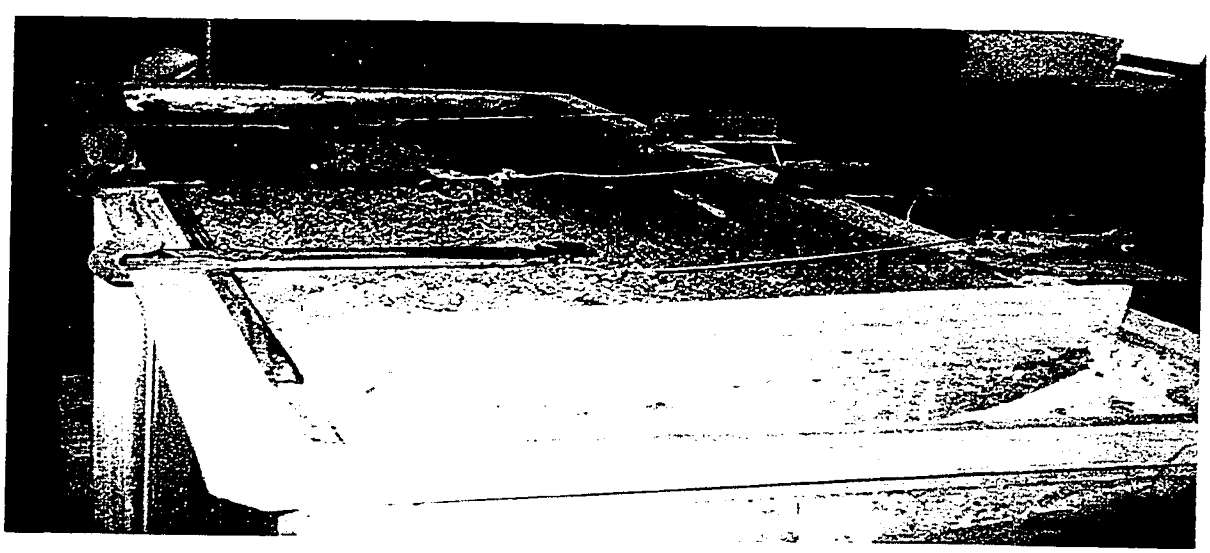

Figure 4.2. Testing Container and the Models of Piles Embedded in the Clayey Soil Before Application of Load.

The clayey soil was prepared by mixing the sand with bentonite clay, the ratio was 1:1 by weight, and then water was added to make it soft clay as described previously in Chapter 3 and then the soil was shovelled into the container and it was compacted by the same manner as previous.

4.4 Driving the Models of the Piles into the Soil and Applying the Lateral Forces

The piles were driven into the soil with a hammer. Three piles were embedded at the same time in the container and were tested simultaneously. Each pile had the same length (400 mm for test in caly, and 350 mm for test in sand) and had different widths. For each test the lateral forces were applied using steel cables. One end of the steel cable was attached to the pile structure 20 mm above the soil surface. The other end of the cable was attached with loading system through the frictionless pulleys and the weights.

4.5 Determination of Lateral Displacement of the Model of the Pile

The lateral displacements were measured at the upper end after the load was applied by measuring the distance between the pile edge and fixed steel bar fixed in the other edge of the container as shown in Fig.4.3.

Displacements of the pile structures were recorder at the point of application of load. Load increments were applied each 10-minute to allow the pile to settle.

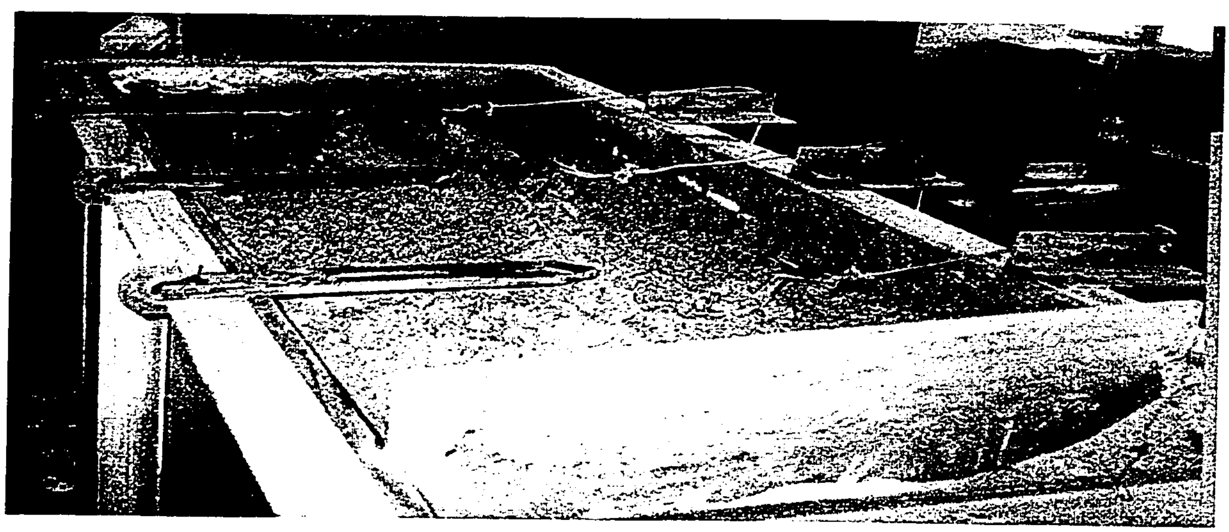

Figure 4.3. Models of the Piles Embedded in Clayey Soil Subjected to Lateral Forces After Failure.

Chapter 5

Statement of the Problem

This chapter is focused on assessment of width - dependent modulus of subgrade reaction $k_h(B)$ of clayey and sandy soil surrounding the models of the piles in laboratory conditions.

The research employs Terzaghi's concept of determination of modulus of subgrade reaction, which is based on static analysis of the pile/soil system. It assumes that in the elastic range the short pile subjected to lateral loading deforms through the rotation. It is commonly accepted fact that for piles embedded in clay soil, the response of the surrounding medium is characterised by constant value of $k_h(B)$ along pile's length. For Winkler foundation the soil reaction is obtained as the product of modulus of subgrade reaction and displacement.

The deformation of short pile through rotation generates soil reaction which has distribution similar to pile's displacement. The intensity of deformation in terms of lateral displacement of a control point depends on the applied horizontal load. The performance of each laterally loaded model of the pile of length $400\,mm$ established by monitoring_the lateral displacement of a chosen control point for all discrete values of lateral force. The clay used in laboratory experiment has unconfined compression strength $q_u = 24\ kPa$ and Young's modulus E_s=1226 kPa determined by means of unconfined compression test.

The investigation of modulus of subgrade reaction of clay involves variable cross sections of the tested models of the piles. The height h of each cross section is the same and is equal to 0.25 *in* (6.35 mm), while widths are determined as product of integer number (from 1 to 8) and its height. The laboratory results in terms of the performance curves are shown in Fig. 5.1. It is worth noting that the force-displacement relationship for each cross section $h \times B_i$ is rather irregular.

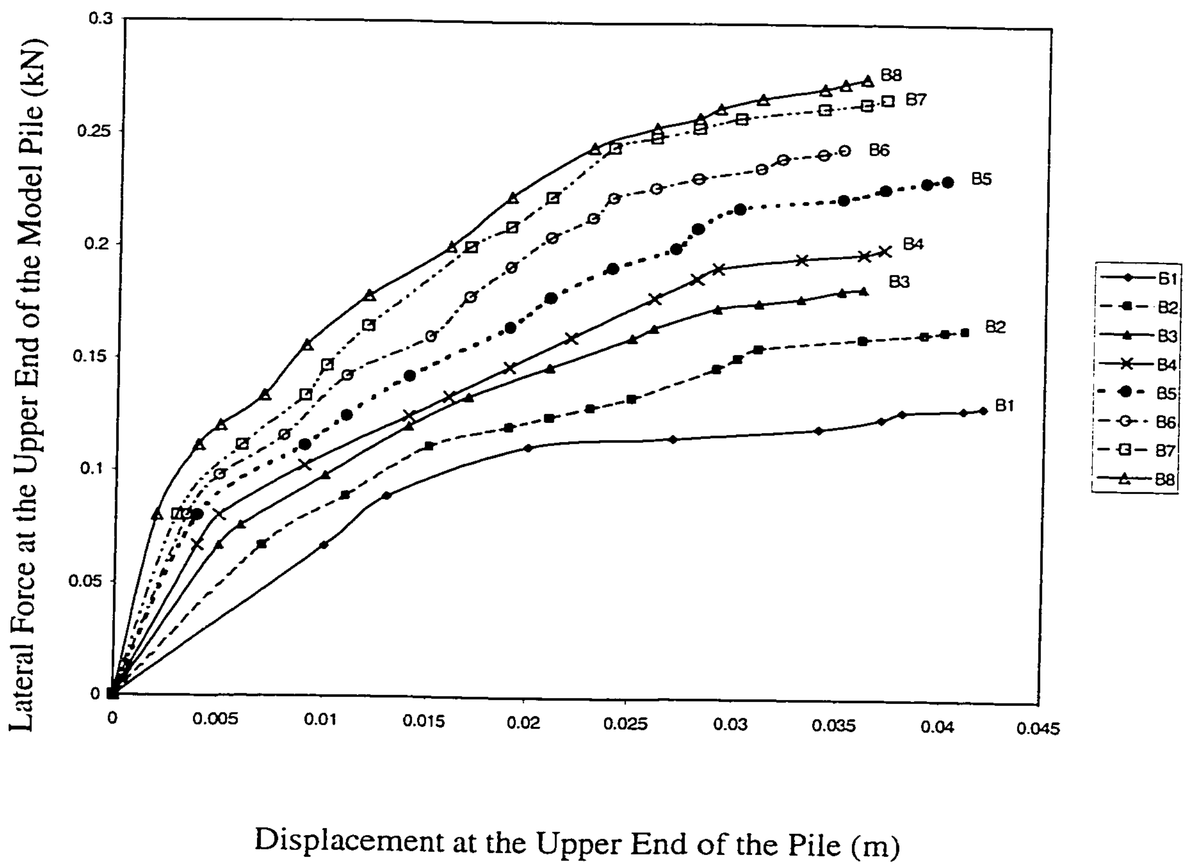

Figure 5.1 Laboratory Force-Displacement Relationship for Laterally Loaded Models of the Pile Having Width B_1, B_2, --- B_8 ($n \times 6.35$ mm) and Embedded in Clay.

Also the investigation of modulus of subgrade reaction of sand involves variable cross sections of the tested models of the piles. The height h of each cross section is the same and is equal to 0.25 *in* (6.35 *mm*), while widths are determined as the product of integer number (from 1 to 8) and its height.

The laboratory results in terms of the performance curves are shown in Fig. 5.2.
It is worth noting that the force-displacement for each cross section $h \times B_i$ is rather irregular.

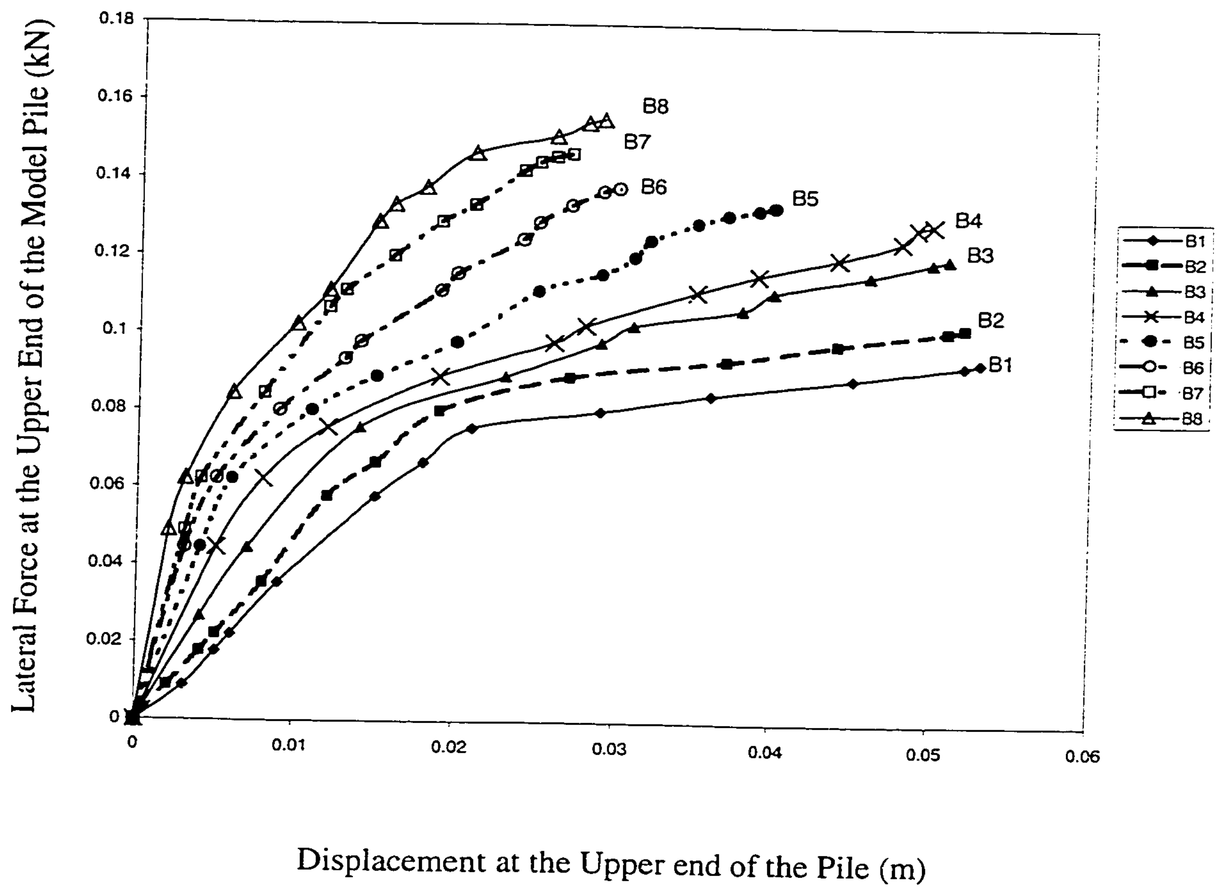

Figure 5.2 Laboratory Force-Displacement Relationship for Laterally Loaded Models of the Pile Having Width $B_1, B_2, \text{---} B_8$ ($n \times 6.35$ mm) and Embedded in Sand.

5.1 Modelling of a Pile as a Beam on Elastic Foundation

The concept of subgrade reaction was introduced into applied mechanics by Winkler (1867), and was used by Zimmermann (1888) for the purpose of computing the stresses in railroad ties, which rest on ballast over their full length. During the following decades the theory was expanded to include the computation of stresses in flexible foundation, such as continuos footings or rafts. The fundamental principle of the theory representing the pile as one dimensional beam element having bending stiffness EI and length L is embedded in homogenous soil. The response of the surrounding soil is represented by systems of springs characterised with constant k_h. By the same concept of Terzaghi the value $k_{hi}B_i$ is constant for any value of B_i. The existing analytical formulation used in investigation of laterally loaded piles suggests that the modulus of subgrade reaction k_h for a given soil is independent of width.

It is demonstrated by the following differential equation of the problem employed in analysis of laterally loaded piles:

$$EI\frac{d^4v}{dz^4} + k_h \cdot B \cdot v = 0 \tag{5.1.1}$$

where:

EI	is the bending stiffness of the pile
z	is current spatial variable along the pile axis
v	is the lateral displacement
k_h	is a horizontal modulus of subgrade reaction (kN/m^3)
B	denotes the width of the pile
$\frac{d^4v}{dz^4}$	is generalised lateral displacement

Eq. (5.1.1) represents the differential equation of the beam on elastic foundation of Winkler type, which is used in analysis of laterally loaded piles in elastic range.

The question, which appears is connected with the second term of Eq. (5.1.1). That is if the Eq. (5.1.1) is correct or should be rather written for the purpose of analysis of laterally loaded piles as:

$$EI\frac{d^4v}{dz^4}+k_h(B)v=0 \tag{5.1.2}$$

where:

$k_h(B)$ is a horizontal modulus of subgrade reaction dependent on the pile's width expressed in (kN/m^2).

According to Terzaghi (1955), the term of subgrade reaction was estimated depending on the value of the p per unit area of the surface of contact between a loaded beam or slab and the subgrade on which it rests on and which it transfers the loads Eq. (5.1.1). The coefficient of vertical subgrade reaction k_s is the ratio between the pressure at any given point of the surface of contact and settlement u produced by loaded application at that point:

$$k_s=\frac{p}{u} \tag{5.1.3}$$

where:

k_s is the vertical modulus of subgrade reaction (kN/m^3)

p is the pressure per unit of area of the surface of contact between a loaded beam and the subgrade (kN/m^2)

u is the vertical settlement (m)

The value of k_s depends on the elastic properties of the subgrade and on the dimensions of the area acted upon the subgrade reaction. In order to illustrate the influence of the width of the beam on the value of vertical modulus of subgrade reaction, the concept of the tube of pressure can conveniently be used. The bulb pressure is arbitrarily defined as the space within which the vertical normal stress in the subgrade is greater than one-fourth of the normal pressure on the surface of load application. The value of one-fourth has been selected because the major portion of the settlement of loaded plate resting on a fairly homogenous subgrade is due to the compression and deformation of the soil located within the space defined by this value. Replacing this value by another one, such as one-third or one-sixth, would have no influence on the conclusions, because the concept of the bulb of pressure merely serves the purpose of assisting to visualise the stress conditions in the loaded subgrade.

Fig.5.3 b shows the bulb of pressure for a beam with B_1 and Fig 5.3 c that for a beam with width nB_1. The depth of these bulbs is D and nD, respectively. The influence of the depth of the bulb on the settlement of the loaded area depends on the deformation characteristics of the subgrade. If the deformation characteristics are more or less independent of depth, it can be assumed that the settlement u increases in simple proportion to the depth of the bulb of pressure:

$$u_n = nu_1 \tag{5.1.4}$$

where:

u_n is the vertical displacement for the beam which has a width = nB_1

u_1 is the vertical displacement for the beam which has a width = B_1

n is arbitrary number denotes the increasing on the width of the beam

So Eq. (5.1.3) yields:

$$k_{sn} = \frac{p}{nu_1} = \frac{p}{u_1}\frac{B_1}{nB_1} \tag{5.1.5}$$

where:

k_{sn} is the vertical modulus of subgrade reaction for the beam which has a width = nB_1.

Substituting $nB_1 = B_i$

$$k_{si} = k_{s1}\frac{B_1}{B_i} \tag{5.1.6}$$

Eq. (5.1.6) leads:

$$k_{si}B_i = k_{s1}B_1 \tag{5.1.7}$$

That is meaning the value of $k_{si}B_i$ is constant for any values of B_i.

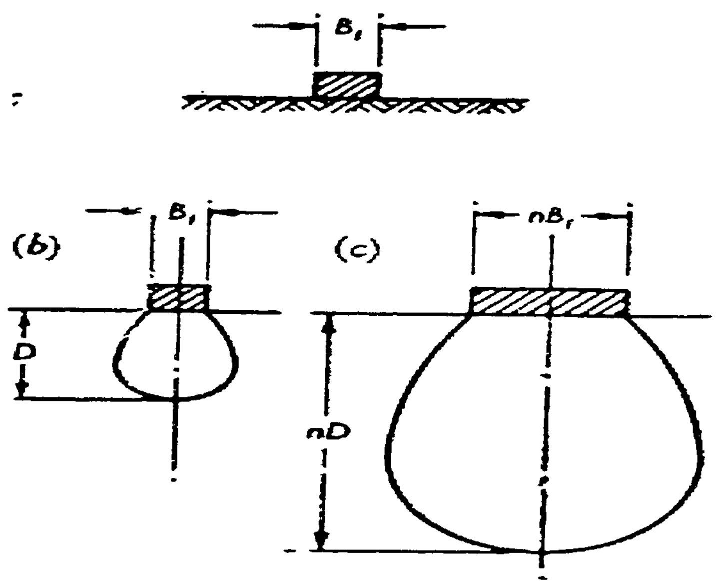

Figure 5.3 Influence of Width of the Applied Pressure on Depth of Bulb of Internal Pressure.

Since about 1920 the theory of subgrade reaction has also been used by several investigators for computing the stresses in piles and sheet piles, which acted upon, by horizontal forces above the ground surface. In this case, the seats of subgrade reaction act in a horizontal direction. Therefore, the ratio between horizontal unit reaction p_h and horizontal displacement v will be referred to as coefficient of horizontal subgrade reaction k_h.

$$k_h = \frac{p_h}{v} \tag{5.1.8}$$

where:

k_h is the vertical modulus of subgrade reaction (kN/m^3)

p_h is the horizontal unit reaction (kN/m^2)

v is the horizontal displacement (m)

In order to investigate the formulated problem the series of laboratory tests for models of the piles of the same length and thickness, however, variable width were tested.

The models of the piles were embedded in homogenous soil and loaded by lateral forces at the upper end of the pile and displacements were recorded for each case.

The determination of modulus of subgrade reaction $k_h(B)$ (for the soils characterized by its constant value for clay as well as varying in linear fashion for sand) was proposed by Terzaghi. Terzaghi's approach is based on static analysis. It postulates that short rigid pile in elastic range subjected to lateral loading deforms through the rigid rotation as shown in Figs. 5.4 and 5.5. Terzaghi's method employed the following assumptions:

1. The laterally loaded pile deforms in linear way through rotation about unknown point o_1 located along the pile axis.
2. The induced soil reaction is of linear type for the soil described by constant value of modulus of subgrade reaction.

3. The induced soil reaction is of parabolic type for the soil described by linearly varying of modulus of subgrade reaction.

The value of n_h(B) for piles driven into sand or of k_h(B) for piles embedded in clay can be determined experimentally. It is done by driving rigid piles with variables widths into the container filled with soil and measuring the horizontal displacement of the upper end of the pile produced by horizontal force acting on the upper end. If the lateral displacement is known, the corresponding values of the coefficient of subgrade reaction can be estimated as described in Sections.5.2 and 5.3.

5.2 Determination of k_h(B) for the Clayey Soil Surrounding Laterally Loaded Piles by Terzaghi's Method

The modulus of subgrade reaction $k_h(B)$ was evaluated for pile with width **B** and depth **H** embedded in clayey soil and subjected to variable lateral force **F.** The displacement at the point of application of force v_T at the upper end of the pile has been measured and the relation between the horizontal subgrade reaction p at any depth z is determined by the equation:

$$p = k_h(B)v \tag{5.2.1}$$

Employing the rigid deformation assumptions, Eq. (5.2.1) can be modified with account for lateral displacement v_T at the upper end of the pile. Thus modified equation of the lateral soil pressure p is as follows:

$$p = k_h(B)v_T \frac{H_2 - z}{H_1 + H_2} \tag{5.2.2}$$

where:

p is the lateral soil reaction at arbitrary point z

$k_h(B)$ is a horizontal modulus of subgrade reaction acting on the model of the pile of width $B\,(kN/m^2)$

v_T is the lateral displacement at the upper end of the pile (m)

v is displacement at arbitrary point z

z is distance from the upper surface of the soil increasing in the pile length direction.

H_1, H_2 and H_3 are geometrical parameters of the model of the pile as shown in the Fig.5.4.

The length of the pile structure.

$$H = H_1 + H_2 + H_3. \tag{5.2.3}$$

The pile is embedded in clayey soil along the depth.

$$D = H_2 + H_3 \tag{5.2.4}$$

H_1 is the part of the pile, which extends above the soil. The pile is loaded by lateral force **F**, which was increased from zero to its final value in incremental fashion (failure of model of the pile is observed, when the small increment load caused large deflection at the top point of the model of the pile).

The analysed the model of the pile satisfies Terzaghi's assumptions, that is:

1. The deformation of the pile is rigid.
2. The deformations are linear.
3. The induced soil reaction/lateral contact pressure has linear distribution.

In the Eq. (5.2.2), the values k_h(B) and H_2 are unknown. They can be computed on the basis of the condition that the sum of horizontal forces and the sum of all moments about an arbitrary chosen point must be equal zero.

In any event the computations furnish the value H_2 of the depth at which point of zero displacement is located.

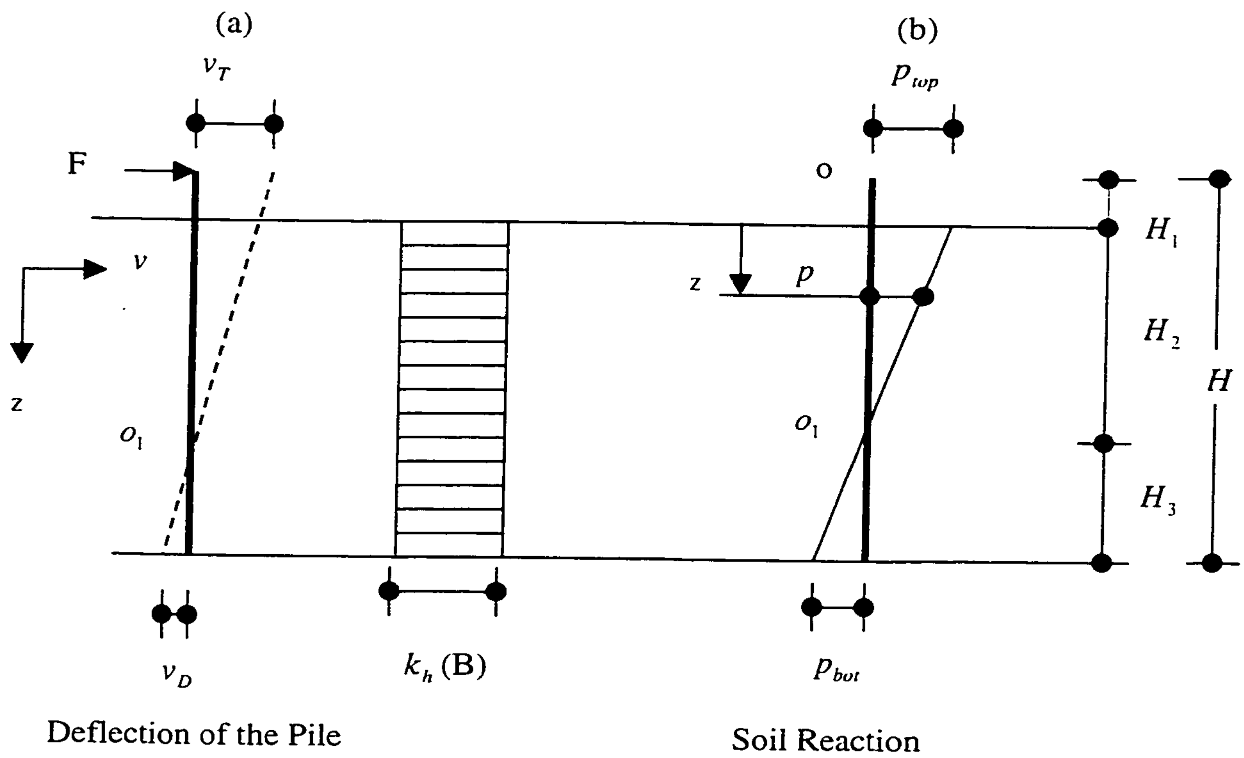

Figure 5.4 Diagram Illustrating Experimental Procedure for Determining Value ($k_h(B)$) of the Model of the Pile Embedded in Clayey Soil.
a) Defection of the Model of the Pile. b) Soil Reaction.

The first equation of static equilibrium is $\sum F_v = 0$

$$F - 0.5 p_{top} H_2 + 0.5 p_{bot} = 0 \tag{5.2.5}$$

The sum of the moments, taken about the pile at the point, where the lateral force is applied should be equal to zero.

The second equation of the equilibrium is $\sum M_O = 0$

$$0.5p_{top}H_2(\frac{H_2}{3}+H_1)-0.5p_{bot}H_3(\frac{2H_3}{3}+H_2+H_1)=0 \tag{5.2.6}$$

To obtain both p_{bot} and p_{top} a variable values of z were substituted in Eq. (5.2.2). They were obtained as follows:

$$p_{bot}=k_h(B)v_T(\frac{-H_3}{H_1+H_2}) \qquad \text{at } z=0 \tag{5.2.7}$$

$$p_{top}=k_h(B)v_T(\frac{H_2}{H_1+H_2}) \qquad \text{at } z=H_2+H_3 \tag{5.2.8}$$

Substitution Eqs. (5.2.7) and (5.2.8) respectively into Eq. (5.2.6) gives:

$$0.5k_h(B)v_T(\frac{H_2}{H_1+H_2})*H_2(\frac{H_2}{3}+H_1)-0.5k_h(B)v_T(\frac{H_3}{H_1+H_2})*H_3(\frac{2H_3}{3}+$$

$$H_2+H_1)=0 \tag{5.2.9}$$

$$\frac{k_h(B)v_T}{2(H_1+H_2)}\left[H_2^{\,2}(\frac{H_2}{3}+H_1)-H_3^{\,2}(\frac{2H_3}{3}+H_2+H_1)\right]=0 \tag{5.2.10}$$

It is worth noting that in Eq. (5.2.10)

$$\frac{k_h(B)v_T}{2(H_1+H_2)}\neq 0 \tag{5.2.11}$$

Thus:

$$H_2^{\,2}(\frac{H_2}{3}+H_1)-H_3^{\,2}(\frac{2H_3}{3}+H_2+H_1)=0 \tag{5.2.12}$$

In the test, the pile's length $H=400\ mm$, and the part of the pile, which extends above the soil $H_1=20\ mm$ giving

$$H_3=H-H_1-H_2 \tag{5.2.13}$$

Substitution of Eq. (5.2.13) in Eq. (5.2.12) results:

$$H_2^{\,2}(\frac{H_2}{3}+H_1)-(H-H_1-H_2)^2(\frac{2}{3}(H-H_1-H_2)+H_2+H_1)=0 \tag{5.2.14}$$

Substituting suitable values for H and H_1 in Eq. (5.2.14) yields:

$$H_2^{\,2}(\frac{H_2}{3}+2)-(40-2-H_2)^2(\frac{2}{3}(38-H_2)+H_2+2)=0 \tag{5.2.15}$$

After simplification of Eq. (5.2.15) through Eqs. (5.2.16), (5.2.17) and (5.2.18). The final equation can be written as in Eq. (5.2.19).

$$\frac{H_2^{\,3}}{3}+2H_2^{\,2}-(38-H_2)^2(27.333-0.3333H_2)=0 \tag{5.2.16}$$

$$\frac{H_2^{\,3}}{3}+2H_2-39468.852+2558.593H_2-52.644H_2^{\,2}+\frac{H_2^{\,3}}{3}=0 \tag{5.2.17}$$

$$\frac{2H_2^{\,3}}{3}-50.644H_2^{\,2}+2558.593H_2-39468.852=0 \tag{5.2.18}$$

$$H_2^{\,3}-76H_2^{\,2}+3837.9H_2-59203.278=0 \tag{5.2.19}$$

By trial and error the distances H_2 and H_3 were obtained as follows:

$$H_2=224.7\ \ mm \tag{5.2.20}$$

$$H_3=155.3\ \ mm \tag{5.2.21}$$

Substitution Eqs. (5.2.7) and (5.2.8) respectively into Eq. (5.2.5) gives:

$$F-0.5k_h(B)v_T(\frac{H_2^{\,2}}{H_1+H_2})+0.5k_h(B)v_T(\frac{H_3^{\,2}}{H_1+H_2})=0 \tag{5.2.21}$$

Substituting the numerical values for H_1, H_2 and H_3 in Eq. (5.2.21) yields:

$$F - 0.5k_h(B)v_T \frac{(0.2247)^2}{(0.2447)^2} + 0.5k_h(B)v_T \frac{(0.1553)^2}{(0.2447)^2} = 0 \qquad (5.2.22)$$

After simplification Eq. (5.2.22) results in:

$$F - 0.0539k_h(B)v_T = 0 \qquad (5.2.23)$$

From Eq. (5.2.23) the horizontal modulus of subgrade reaction was obtain as follows:

$$k_h(B) = 18.55\frac{F}{v_T} \qquad (kN/m^2) \qquad (5.2.24)$$

Consequently the modulus of subgrade reaction expressed in (kN/m^3) results:

$$k_h = 18.55\frac{F}{v_T \cdot B} \qquad (kN/m^3) \qquad (5.2.25)$$

The modulus of subgrad reaction in both of Eqs. (5.2.24) and (5.2.25) are dependent on the lateral force and the lateral displacement at the upper end of the pile.

5.3 Determination of n_h(B) for the Sandy Soil Surrounding Laterally Loaded Piles by Terzaghi's Method

The procedure for determination the constant of horizontal subgrade reaction n_h(B) for models of the pile embedded in sandy soil as shown in the Fig. 5.4. The laterally loaded models of the piles subjected to horizontal forces are similar these in clayey soil as described earlier. Consider now a pile structure with length embedded

$$H = H_1 + H_2 + H_3. \qquad (5.3.1)$$

The pile is embedded in sandy soil along the depth

$$D = H_2 + H_3. \qquad (5.3.2)$$

H_1 is the part of the pile, which extends above the soil. The pile is loaded by lateral force **F**, which increases from zero to its final value (failure of model of the pile is observed, when the small increment load caused large deflection at the top point of the model of the pile).

The analysed pile satisfies Terzaghi's assumptions, that is:

1. The deformation of the pile is through rotation.
2. The deformations are linear.
4. The induced soil reaction/lateral contact pressure has linear distribution.

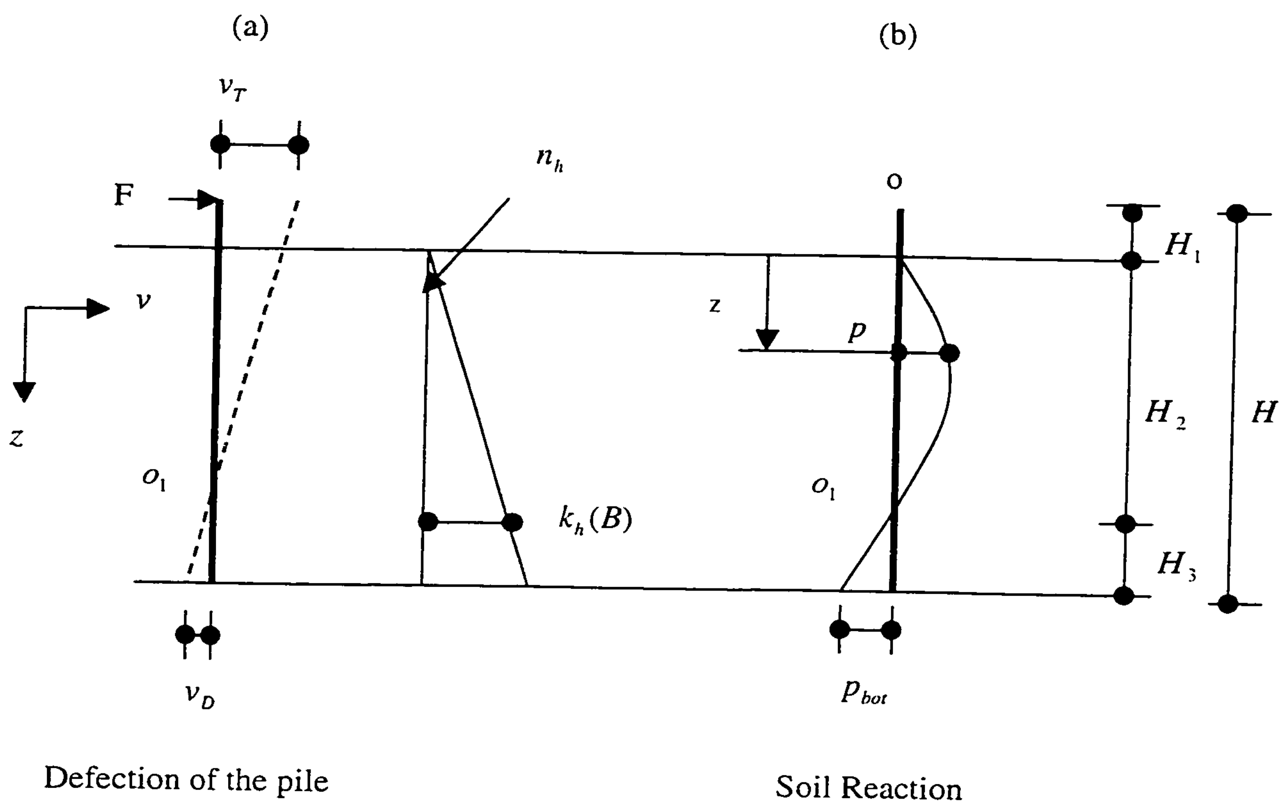

Figure 5.5 Diagram Illustrating Experimental Procedure for Determining Value of the Model of the Pile Embedded in Sandy Soil.
a) Defection of the Model of the Pile. b) Soil Reaction.

The displacement v_T of the upper end of the pile has been measured and the relation between the horizontal subgrade reaction p at any depth z is determined employing Eq. (5.2.1) presented below.

Thus:

$$p = k_h(B)v \tag{5.2.1}$$

In cohesionless soil, such as clean sand, the soil reaction p required to produce a given displacement v increases approximately in simple proportion to depth z. Thus:

$$k_h(B) = \frac{p}{v} = n_h(B)z \tag{5.3.3}$$

Substituting Eq. (5.3.3) in Eq. (5.2.1), results:

$$p = n_h(B)zv \tag{5.3.4}$$

Employing the rigid deformation assumptions (deformation behaves in linear fashion), Eq. (5.2.2) can be modified with account for lateral displacement v_T at the upper end of the pile. Thus Eq. (5.2.2) can be written as follows:

$$p = n_h(B)zv_T \frac{H_2 - z}{H_1 + H_2} \tag{5.3.5}$$

where:

p is the lateral soil reaction at arbitrary point z.

$k_h(B)$ is a horizontal modulus of subgrade reaction expressed in (kN/m^2).

$n_h(B)$ is a constant of horizontal subgrade reaction expressed in (kN/m^3).

v_T is the lateral displacement of the upper end of the pile expressed in (m).

v is the lateral displacement at arbitrary point z.

z is the distance from the upper surface of the soil increasing in the pile length direction.

H_1, H_2 and H_3 are geometrical parameters of the pile as shown in the Fig.5.4.

The first equation of equilibrium $\sum F_v = 0$.

$$F - \frac{2}{3} p_{\max} H_2 + \frac{1}{3} p_{bot} H_3 = 0 \qquad (5.3.6)$$

The second equation of the equilibrium is $\sum M_O = 0$

$$(\frac{2}{3} p_{\max} H_2)(H_1 + \frac{H_2}{2}) - (\frac{1}{3} p_{bot} H_3)(H_1 + H_2 + \frac{3}{4} H_3) = 0 \qquad (5.3.7)$$

Substitution in Eq. (5.3.5) for

$$z = H_2 + H_3 \qquad (5.3.8)$$

Thus:

p_{bot} can be determined as:

$$p_{bot} = \frac{n_h(B)v_T}{(H_1 + H_2)}(H^2 - 2HH_1 + H_1^{\ 2} - HH_2 + H_1 H_2) \qquad (5.3.9)$$

Insertion Eq. (5.2.3) into Eq. (5.3.9), gives:

$$p_{bot} = \frac{n_h v_T}{(H_1 + H_2)}(H_3^{\ 2} + H_2 H_3) \qquad (5.3.10)$$

It is worth noting that $p_{\max}$ can be determined by substitution in Eq. (5.3.3) the relationship $z = \frac{H_2}{2}$ (5.3.11)

which results:

$$p_{\max} = \frac{n_h v_T}{(H_1 + H_2)} * \frac{H_2^{\ 2}}{4} \qquad (5.3.12)$$

Substitution Eqs (5.3.10) and (5.3.12) into Eq (5.3.7) gives:

$$\frac{2}{3}*\frac{n_h v_T}{(H_1+H_2)}*\frac{H_2^{\ 2}}{4}*H_2(H_1+\frac{H_2}{2})-(\frac{1}{3}*\frac{n_h v_T}{(H_1+H_2)}(H^2-2HH_1+H_1^{\ 2}-HH_2+$$

$$H_1H_2)(H-H_1-H_2)(H_1+H_2+\frac{3H_3}{4}))=0 \qquad (5.3.13)$$

After simplification Eq. (5.3.13) through two steps in Eqs. (5.3.14) and (5.3.15), Eq. (5.3.13) can be written in form as Eq. (5.3.16):

$$\frac{n_h v_T}{3}[(\frac{H_2^{\ 3}(H_1+0.5H_2)}{2(H_1+H_2)}-\frac{(H^2-2HH_1+H_1^{\ 2}-HH_2+H_1H_2)(H-H_1-H_2)}{(H_1+H_2)}*$$

$$(H_1+H_2+\frac{3}{4}(H-H_1-H_2))=0 \qquad (5.3.14)$$

$$\frac{n_h v_T}{3(H_1+H_2)}\neq 0 \qquad (5.3.15)$$

$$\frac{H_2^{\ 3}}{2}(H_1+0.5H_2)-(H^2-2HH_1+H_1^{\ 2}-HH_2+H_1H_2)(H-H_1-H_2)*$$

$$(H_1+H_2+\frac{3}{4}(H-H_1-H_2))=0 \qquad (5.3.16)$$

In the test, the pile's length $H=350\ mm$, and the part of the pile, which extends above the soil $H_1=17.5\ mm$.

Substitution of Eq. (5.2.13) in Eq. (5.3.16) results:

$$\frac{H_2^{\ 3}}{2}(1.75+0.5H_2)-(35^2-2*35*1.75+1.75^2-35H_2+1.75H_2)(35-1.75-H_2)*$$

$$(1.75+H_2+0.75(35-1.75-H_2))=0 \qquad (5.3.17)$$

After simplification of Eq. (5.3.17) through Eqs. (5.3.18), (5.3.19), (5.3.20), and (5.3.21), the final equation can be written as Eq. (5.3.22).

$$0.875H_2^3 + 0.25H_2^4 - (1105.563 - 33.25H_2)(33.25 - H_2)(1.75 + H_2 + 0.75(33.25 - H_2)) = 0 \tag{5.3.18}$$

$$0.875H_2^3 + 0.25H_2^4 - (1105.563 - 33.25H_2)(33.25 - H_2)(26.688 + 0.25H_2) = 0 \tag{5.3.19}$$

$$0.875H_2^3 + 0.25H_2^4 - 981050.88 + 49820.531H_2 - 334.6H_2^2 - 8.3125H_2^3 = 0 \tag{5.3.20}$$

$$0.25H_2^4 - 7.4375H_2^3 - 334.6H_2^2 + 49820.531H_2 - 392403.52 = 0 \tag{5.3.21}$$

$$H_2^4 - 29.75H_2^3 - 1338.4H_2^2 + 199282.124H_2 - 392403.52 = 0 \tag{5.3.22}$$

By trial and error the distances H_2 and H_3 were obtained as follows:

$$H_2 = 239.5\ mm \tag{5.3.23}$$

$$H_3 = 93\ \ mm \tag{5.3.24}$$

Substitution Eqs. (5.3.10) and (5.3.12)into Eq. (5.3.6) gives:

$$F - \frac{2}{3} * \frac{n_h(B)v_T}{(H_1 + H_2)} * \frac{H_2}{4} + \frac{1}{3} * \frac{n_h(B)v_T}{(H_1 + H_2)}\ (H^2 - 2HH_1 + H_1^2 - HH_2 + H_1H_2) * H_3 = 0 \tag{5.3.25}$$

After simplification of Eq. (5.3.25) results:

$$F - \frac{1}{3} * \frac{n_h(B)v_T}{(H_1 + H_2)}(\frac{H_2^3}{2} + (H^2 - 2HH_1 + H_1^2 - HH_2 + H_1H_2)) * H_3 = 0 \tag{5.3.26}$$

Substitution the values of H_2 and H_3 given by Eq. (5.3.23) and (5.3.24) respectively into Eq. (5.3.26) gives:

$$F - \frac{n_h(B)v_T}{3(0.0175+0.2395)}(\frac{0.2395^2}{2} + 0.35^2 - 2*0.35*0.0175 + 0.0175^2 - 0.35*0.2395 +$$

$$0.0175*0.2395)*0.093 = 0 \qquad (5.3.27)$$

After simplification of Eq. (5.3.27) results:

$$F - \frac{0.0035 n_h(B)v_T}{3*0.257} = 0 \qquad (5.3.28)$$

From Eq. (5.3.28) the constant of horizontal modulus of subgrade reaction $n_h(B)$ was obtained as follows:

$$n_h(B) = \frac{F}{0.004539 v_T} \qquad (5.3.29)$$

Eq. (5.3.29) can be written in the following form:

$$n_h(B) = 220.286 \frac{F}{v_T} \quad (kN / m^3) \qquad (5.3.30)$$

It is worth noting that the constant of modulus of subgrade reaction expressed in (kN / m^4) can be written in following form:

$$n_h = 220.286 \frac{F}{v_T \cdot B} \quad (kN / m^4) \qquad (5.3.31)$$

5.4 Determination of k_h(B) for the Clayey Soil Surrounding Laterally Loaded Piles by Vesic's Method

This method for determination the horizontal modulus of subgrade reaction $k_h(B)$ is proposed by Vesic. Vesic's formula as shown in Eq. (5.4.1) depends on the width of the pile and properties of both pile and soil.

$$k(B) = 0.65 * \sqrt[12]{\frac{E_s B^4}{E_p I_P}} \cdot \frac{E_s}{1-\mu_s^{\,2}} \tag{5.4.1}$$

where:

$k_h(B)$ is a horizontal modulus of subgrade reaction expressed in (kN/m^2)

E_S is modulus of elasticity of soil expressed in (kPa)

E_p is modulus of elasticity of pile expressed in (kPa)

I_p is moment of inertia of the pile $= \dfrac{Bh^3}{12}$ (m^4)

B is the width of the pile (m)

μ_s is Poisson's ratio

5.4.1 Determination of $k_h(B)$ for Soil Type 1

The soil type 1 consists of 50% sand plus 50% nontreated bentonite clay and 2%lime that was added to the soil to make the soil more brittle. This soil has modulus of elasticity E_s= 1226 kPa.

The modulus of elasticity of the model of the pile which driven into the soil E_p = 200e06 kPa and $\mu_s = 0.4$

- For cross section $B_1 = 6.35\ mm$

The moment of inertia of the model of the pile was determined as follows:

$$I_p = \frac{B_1 h^3}{12} = \frac{0.00635 \cdot (0.00635)^3}{12} = 1.355e-10\ \ m^4 \tag{5.4.2}$$

The modulus of subgrade reaction was determined by employing Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.00635)^4}{200e06 * 1.355e-10}} \cdot \frac{1226}{1-0.4^2} = 429\ \ kN/m^2 \tag{5.4.3}$$

- For cross section $B_2 = 12.7\ mm$

The moment of inertia of the model of the pile was determined by the following relationship:

$$I_p = \frac{B_2 h^3}{12} = \frac{0.0127 \cdot (0.00635)^3}{12} = 2.71e-10\ m^4 \tag{5.4.4}$$

The modulus of subgrade reaction was evaluated by employing Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.0127)^4}{200e06 * 2.71e-10}} \cdot \frac{1226}{1-0.4^2} = 510\ kN/m^2 \tag{5.4.5}$$

- For cross section $B_3 = 19.05\ mm$

The moment of inertia of the model of the pile was calculated in accordance with:

$$I_p = \frac{B_3 h^3}{12} = \frac{0.01905 \cdot (0.00635)^3}{12} = 4.06e-10\ m^4 \tag{5.4.6}$$

The modulus of subgrade reaction was calculated by employing Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.01905)^4}{200e06 * 4.06e-10}} \cdot \frac{1226}{1-0.4^2} = 565\ \ kN/m^2 \tag{5.4.7}$$

- For cross section $B_4 = 25.4\ mm$

The moment of inertia of the model of the pile was evaluated as:

$$I_p = \frac{B_4 h^3}{12} = \frac{0.0254 \cdot (0.00635)^3}{12} = 5.42e-10\ \ m^4 \tag{5.4.8}$$

The modulus of subgrade reaction was obtained by using Eq. (5.4.1) that results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.0254)^4}{200e06 * 5.42e-10}} \cdot \frac{1226}{1-0.4^2} = 607\ kN/m^2 \tag{5.4.9}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was obtained as follows:

$$I_p = \frac{B_h h^3}{12} = \frac{0.03175 \cdot (0.00635)^3}{12} = 6.77e-10\ m^4 \tag{5.4.10}$$

The modulus of subgrade reaction was determined by employing Eq. (5.4.1), which gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.03175)^4}{200e06 * 6.77e-10}} \cdot \frac{1226}{1-0.4^2} = 642\ kN/m^2 \tag{5.4.11}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was evaluated in accordance with:

$$I_p = \frac{B_6 h^3}{12} = \frac{0.0381 \cdot (0.00635)^3}{12} = 8.13e-10\ m^4 \tag{5.4.12}$$

The modulus of subgrade reaction was obtained by using Eq. (5.4.1), which results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.0381)^4}{200e06 * 8.13e-10}} \cdot \frac{1226}{1-0.4^2} = 672\ kN/m^2 \tag{5.4.13}$$

- For cross section $B_7 = 44.45\ mm$

The moment of inertia of the model of the pile was determined as follows:

$$I_p = \frac{B_7 h^3}{12} = \frac{0.04445 \cdot (0.00635)^3}{12} = 9.48e-10\ m^4 \tag{5.4.14}$$

The modulus of subgrade reaction was calculated by employing Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.04445)^4}{200e06 * 9.48e-10}} \cdot \frac{1226}{1-0.4^2} = 698\ kN/m^2 \tag{5.4.15}$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was obtained as:

$$I_p = \frac{B_8 h^3}{12} = \frac{0.0508 \cdot (0.00635)^3}{12} = 10e-10\ m^4 \tag{5.4.16}$$

The modulus of subgrade reaction was evaluated by using Eq. (5.4.1), which results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1226 * (0.0508)^4}{200e06 * 10e-10}} \cdot \frac{1226}{1-0.4^2} = 727\ kN/m^2 \tag{5.4.17}$$

5.4.2 Determination of $k_h(B)$ for Soil Type 2

The soil type 2 consists of 40% sand plus 60% nontreated bentonite clay and 2%lime was added to the soil to make the soil more brittle. This soil has modulus of elasticity $E_s = 1486\ kPa$.

The modulus of elasticity of the model of the pile, which driven into the soil $E_p =$ 200e06 kPa and $\mu_s = 0.4$

- For cross section $B_1 = 6.35\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.2) as follows:

I_p = 1.335e-10 m^4

The modulus of subgrade reaction was determined by employing Eq. (5.4.1), which gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486 * (0.00635)^4}{200e06 * 1.355e-10}} \cdot \frac{1486}{1-0.4^2} = 529\ kN/m^2 \tag{5.4.18}$$

- For cross section $B_2 = 12.7\ mm$

The moment of inertia of the model of the pile was evaluted earlier by means of Eq. (5.4.4) as:

I_p = 2.71e-10 m^4

The modulus of subgrade reaction was evaluated by using Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486 * (0.0127)^4}{200e06 * 2.71e-10}} \cdot \frac{1486}{1-0.4^2} = 629 \ kN/m^2 \tag{5.4.19}$$

- For cross section B_3=19.05 mm

The moment of inertia of the model of the pile was calculated by using Eq. (5.4.6), which gives:

I_p =4.06e-10 m^4

The modulus of subgrade reaction was obtained by means of Eq. (5.4.1) that results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486 * (0.01905)^4}{200e06 * 4.06e-10}} \cdot \frac{1486}{1-0.4^2} = 696 \ kN/m^2 \tag{5.4.20}$$

- For cross section B_4= 25.4 mm

The moment of inertia of the model of the pile was obtained formerly in Eq. (5.4.8) as:

I_p =5.42e-10 m^4

The modulus of subgrade reaction was evaluated by using Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486 * (0.0254)^4}{200e06 * 5.42e-10}} \cdot \frac{1486}{1-0.4^2} = 748 \ kN/m^2 \tag{5.4.21}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was determined earlier by means of Eq. (5.4.10) as follows:

I_p =6.77e-10 m^4

The modulus of subgrade reaction was calculated by employing Eq. (5.4.1), which gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486*(0.03175)^4}{200e06*6.77e-10}} \cdot \frac{1486}{1-0.4^2} = 791\ kN/m^2 \tag{5.4.22}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was evaluated formerly by using Eq. (5.4.12), which gives:

I_p =8.13e-10 m^4

The modulus of subgrade reaction was determined by using Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486*(0.0381)^4}{200e06*8.13e-10}} \cdot \frac{1486}{1-0.4^2} = 828\ kN/m^2 \tag{5.4.23}$$

- For cross section $B_7 = 44.45\ mm$

The moment of inertia of the model of the pile was calculated previously by means of Eq. (5.4.14) as:

I_p =9.48e-10 m^4

The modulus of subgrade reaction was obtained by employing Eq. (5.4.1) that results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486*(0.04445)^4}{200e06*9.48e-10}} \cdot \frac{1486}{1-0.4^2} = 860 kN/m^2\ kN/m^2 \qquad (5.4.24)$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was evaluated earlier by using Eq. (5.4.14) as follows:

I_p = 10e-10 m^4

The modulus of subgrade reaction was obtained by employing Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1486*(0.0508)^4}{200e06*10e-10}} \cdot \frac{1486}{1-0.4^2} = 895\ kN/m^2 \qquad (5.4.25)$$

5.4.3 Determination of $k_h(B)$ for soil type 3

The soil type 3 consists of 25% sand plus 75% nontreated bentonite and 2%lime, which was added to the soil to make the soil more brittle. This soil has modulus of elasticity $E_s = 1367\ kPa$.

The modulus of elasticity of the model of the pile, which driven into the soil $E_p =$ 200e06 kPa and $\mu_s = 0.4$.

- For cross section $B_1 = 6.35\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.2) as follows:

I_p = 1.335e-10 m^4

The modulus of subgrade reaction was obtained by employing Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.00635)^4}{200e06 * 1.355e-10}} \cdot \frac{1367}{1-0.4^2} = 483\ kN/m^2 \tag{5.4.26}$$

- For cross section B_2= 12.7 mm

The moment of inertia of the model of the pile was obtained formerly in Eq. (5.4.4) as:

I_p = 2.71e-10 m^4

The modulus of subgrade reaction was calculated by using Eq. (5.4.1), which results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.0127)^4}{200e06 * 2.71e-10}} \cdot \frac{1367}{1-0.4^2} = 574\ kN/m^2 \tag{5.4.27}$$

- For cross section B_3= 19.05 mm

The moment of inertia of the model of the pile was calculated earlier in Eq. (5.4.6) as:

I_p =4.06e-10 m^4

The modulus of subgrade reaction was evaluated by means of Eq. (5.4.1) that results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.01905)^4}{200e06 * 4.06e-10}} \cdot \frac{1367}{1-0.4^2} = 636\ kN/m^2 \tag{5.4.28}$$

- For cross section B_4=25.4 mm

The moment of inertia of the model of the pile was obtained previously in Eq. (5.4.8) as follows:

I_p =5.42e-10 m^4

The modulus of subgrade reaction was calculated by using Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.0254)^4}{200e06 * 5.42e-10}} \cdot \frac{1367}{1-0.4^2} = 683\ kN/m^2 \tag{5.4.29}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was calculated formerly in Eq. (5.4.10) as fol:

I_p =6.77e-10 m^4

The modulus of subgrade reaction was obtained by employing Eq. (5.4.1) that gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.03175)^4}{200e06 * 6.77e-10}} \cdot \frac{1367}{1-0.4^2} = 722\ kN/m^2 \tag{5.4.30}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was evaluated earlier by means of Eq. (5.4.12) as:

I_p =8.13e-10 m^4

The modulus of subgrade reaction was obtained by employing Eq. (5.4.1) as follows:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.0381)^4}{200e06 * 8.13e-10}} \cdot \frac{1367}{1-0.4^2} = 756\ kN/m^2 \tag{5.4.31}$$

- For cross section $B_7 = 44.45\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.14) as:

I_p =9.48e-10 m^4

The modulus of subgrade reaction was evaluated by using Eq. (5.4.1) that results:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.04445)^4}{200e06 * 9.48e-10}} \cdot \frac{1367}{1-0.4^2} = 786\ kN/m^2 \tag{5.4.32}$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was obtained earlier in Eq. (5.4.14) as follows:

$$I_p = 10\text{e-}10\ m^4$$

The modulus of subgrade reaction was evaluated by employing Eq. (5.4.1), which gives:

$$k(B) = 0.65 * \sqrt[12]{\frac{1367 * (0.0508)^4}{200e06 * 10e-10}} \cdot \frac{1367}{1-0.4^2} = 818\ kN/m^2 \qquad (5.4.33)$$

5.5. Calculation of the Characteristic Pile Length λ and Pile Length L for the Clayey Soil Surrounding Laterally Loaded Piles Employing Vesic's Method

To evaluate the length of the model of the pile, which using in the test and to be sure the model of the pile is short, characteristic length of the model of the pile λ is necessary to estimate the length of the pile. The characteristic length λ of the pile structure with bending stiffness $E_p I_p$ embedded in soil with $k_h(B)$ is constant is given by the following formula:

$$\lambda = \sqrt[4]{\frac{E_p I_p}{k_h(B)}} \qquad (5.5.1)$$

The following condition must be satisfied for short pile:

$$\frac{L}{\lambda} \le 5 \qquad (5.5.2)$$

$$L \le 5\lambda \qquad (5.5.3)$$

5.5.1 Determination of λ for Soil Type 1

The properties of soil type 1 and the model of the pile were described in sec.5.4.1

- For cross section $B_1 = 6.35\ mm$

The moment of inertia of the model of the pile was determined previously by means of Eq. (5.4.2) as follows:

I_p = 1.335e-10 m^4

The modulus of subgrade reaction was determined in Eq. (5.4.3) as:

$k_h(B) = 429\ \ kN/m^2$

The characteristic length of the pile structure was obtained by employing Eq. (5.5.1), which gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 1.355e-10}{429}} = 0.089\ m \tag{5.5.4}$$

The maximum length of the short pile was calculated using Eq. (5.5.3) that results:

$$L_{MAX} = 5 * 0.089 = 0.446\ \ m \tag{5.5.5}$$

- For cross section $B_2 = 12.7\ mm$

The moment of inertia of the model of the pile was evaluated previously in Eq. (5.4.4) as follows:

$I_p = 2.71e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.5) as:

$k_h(B) = 510\ kN/m^2$

The characteristic length of the pile structure was determined by employing Eq. (5.5.1) that gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 2.71e-10}{510}} = 0.1015\ m \tag{5.5.6}$$

The maximum length of the short pile was obtained by using Eq. (5.5.3), which results:

$$L_{MAX} = 5 * 0.1015 = 0.5076\ m \tag{5.5.7}$$

- For cross section $B_3 = 19.05\,mm$

The moment of inertia of the model of the pile was calculated earlier in Eq. (5.4.6) as:

$$I_p = 4.06e-10\ m^4$$

The modulus of subgrade reaction was obtained by means of Eq. (5.4.7) that results:

$$k_h(B) = 565\ kN/m^2$$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1) as:

$$\lambda = \sqrt[4]{\frac{200e6 * 4.06e-10}{565}} = 0.1095\ m \tag{5.5.8}$$

The maximum length of the short pile was obtained by employing Eq. (5.5.3) that gives:

$$L_{MAX} = 5 * 0.1095 = 0.5475\ m \tag{5.5.9}$$

- For cross section $B_4 = 25.4\ mm$

The moment of inertia of the model of the pile was determined earlier in Eq. (5.4.8) as follows:

$I_p = 5.42e-10\ m^4$

The modulus of subgrade reaction was calculated in Eq. (5.4.9), which results:

$k_h(B) = 607\ kN/m^2$

The characteristic length of the pile structure was obtained by employing Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 5.42e-10}{607}} = 0.1156\ m \tag{5.5.10}$$

The maximum length of the short pile was calculated by using Eq. (5.5.3), which gives:

$$L_{MAX} = 5 * 0.1156 = 0.578\ m \tag{5.5.11}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was determined formerly in Eq. (5.4.10) as:

$I_p = 6.77e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.11) as follows:

$k_h(B) = 642\ kN/m^2$

The characteristic length of the pile structure was determined by employing Eq. (5.5.1) that results:

$$\lambda = \sqrt[4]{\frac{200e6 * 6.77e-10}{642}} = 0.12\ m \tag{5.5.12}$$

The maximum length of the short pile was calculated by using Eq. (4.5.3) in accordance with:

$$L_{MAX} = 5 * 0.12 = 0.6\ m \tag{5.5.13}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.12) as follows:

$I_p = 8.13e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.13) as:

$k_h(B) = 672\ kN/m^2$

The characteristic length of the pile structure was calculated by employing Eq. (5.5.1) that results:

$$\lambda = \sqrt[4]{\frac{200e6 * 8.13e-10}{672}} = 0.125\ m \tag{5.5.14}$$

The maximum length of the short pile was obtained by using Eq. (5.5.3), which gives:

$$L_{MAX} = 5 * 0.125 = 0.625\ m \tag{5.5.15}$$

- For cross section $B_7 = 44.45\ mm$

The moment of inertia of the model of the pile was calculated formerly in Eq. (5.4.14) as:

$I_p = 9.48e-10\ m^4$

The modulus of subgrade reaction was evaluted in Eq. (5.4.15), which results:

$k_h(B) = 698\ kN/m^2$

The characteristic length of the pile structure was determined by employing Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 9.48e-10}{698}} = 0.128\ m \qquad (5.5.16)$$

The maximum length of the short pile was obtained by using Eq. (5.5.3) that results:

$$L_{MAX} = 5 * 0.128 = 0.64\ m \qquad (5.5.17)$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was determined earlier in Eq. (5.4.16) that results:

$I_p = 10e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.17) as:

$k_h(B) = 727\ kN/m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 10e-10}{727}} = 0.131\ m \qquad (5.5.18)$$

The maximum length of the short pile was determined employing Eq. (5.5.3) as follows:

$$L_{MAX} = 5 * 0.131 = 0.655\ m \qquad (5.5.19)$$

5.5.2 Determination of λ for Soil Type 2

The properties of soil type 1 and the model of the pile were described in Sec.5.4.2

- For cross section $B_1 = 6.35\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.2) as follows:

I_p = 1.335e-10 m^4

The modulus of subgrade reaction was obtained in Eq. (5.4.18) as:

$k_h(B) = 529\ kN/m^2$

The characteristic length of the pile structure was calculated by employing Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 1.355e-10}{529}} = 0.0846\ m \qquad (5.5.20)$$

The maximum length of the short pile was evaluated by using Eq. (5.5.3), which gives:

$$L_{MAX} = 5 * 0.0846 = 0.423\ m \qquad (5.5.21)$$

- For cross section $B_2 = 12.7\ mm$

The moment of inertia of the model of the pile was determined earlier in Eq. (5.4.4) as:

$I_p = 2.71e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.19) as follows:

$k_h(B) = 629\ kN/m^2$

The characteristic length of the pile structure was calculated by means of Eq. (5.5.1) as:

$$\lambda = \sqrt[4]{\frac{200e6 * 2.71e-10}{629}} = 0.096\ m \tag{5.5.22}$$

The maximum length of the short pile was obtained by employing Eq. (4.5.3), which results:

$$L_{MAX} = 5 * 0.096 = 0.4817\ m \tag{5.5.23}$$

- For cross section $B_3 = 19.05\,mm$

The moment of inertia of the model of the pile was evaluated formerly in Eq. (5.4.6) as:

$$I_p = 4.06e-10\ m^4$$

The modulus of subgrade reaction was obtained in Eq. (5.4.20) that results:

$$k_h(B) = 696\ kN/m^2$$

The characteristic length of the pile structure was calculated by employing Eq. (5.5.1), which gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 4.06e-10}{696}} = 0.1038\ m \tag{5.5.24}$$

The maximum length of the short pile was determined by means of Eq. (5.5.3) as follows:

$$L_{MAX} = 5 * 0.1038 = 0.52\ m \tag{5.5.25}$$

- For cross section $B_4 = 25.4\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.8) as:

$I_p = 5.42e - 10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.21) that results:

$k_h(B) = 748\ kN / m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1), which gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 5.42e-10}{748}} = 0.1097\ m \tag{5.5.26}$$

The maximum length of the short pile was evaluated by employing Eq. (5.5.3) as follows:

$$L_{MAX} = 5 * 0.1097 = 0.55\ m \tag{5.5.27}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was evaluated formerly in Eq. (5.4.10) as:

$I_p = 6.77e - 10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.22), which results:

$k_h(B) = 791\ kN / m^2$

The characteristic length of the pile structure was determined by means of Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 6.77e-10}{791}} = 0.1144\ m \tag{5.5.28}$$

The maximum length of the short pile was determined by using Eq. (5.5.3) that results:

$$L_{MAX} = 5 * 0.1144 = 0.572\ m \tag{5.5.29}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.12) as follows:

$I_p = 8.13e-10\ m^4$

The modulus of subgrade reaction was calculated in Eq. (5.4.23), which results:

$k_h(B) = 828\ kN/m^2$

The characteristic length of the pile structure was obtained by employing Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 8.13e-10}{828}} = 0.1184\ m \tag{5.5.30}$$

The maximum length of the short pile was calculated by using Eq. (5.5.3) that results:

$$L_{MAX} = 5 * 0.1184 = 0.592\ m \tag{5.5.31}$$

- For cross section $B_7 = 44.45\,mm$

The moment of inertia of the model of the pile was obtained earlier in Eq. (5.4.14) as:

$I_p = 9.48e-10\ m^4$

The modulus of subgrade reaction was determined in Eq. (5.4.24), which results:

$k_h(B) = 860\ kN/m^2$

The characteristic length of the pile structure was calculated by means of Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 9.48e-10}{860}} = 0.1218\ m \qquad (5.5.32)$$

The maximum length of the short pile was evaluated by employing Eq. (5.5.3) as follows:

$$L_{MAX} = 5 * 0.1218 = 0.61m \qquad (5.5.33)$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.16) as follows:

$I_p = 10e-10\ m^4$

The modulus of subgrade reaction was obtained in Eq. (5.4.25), which gives:

$k_h(B) = 895\ kN/m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 10e-10}{895}} = 0.1222\ m \qquad (5.5.34)$$

The maximum length of the short pile was obtained by employing Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.1222 = 0.6113\ m \qquad (5.5.35)$$

5.5.3 Determination of λ for soil type 3

The properties of soil type 3 and the model of the pile were described in sec.5.4.3

- For cross section $B_1 = 6.35\,mm$

The moment of inertia of the model of the pile was determined previously in Eq. (5.4.2) as follows:

I_p = 1.335e-10 m^4

The modulus of subgrade reaction was obtained in Eq. (5.4.26) that results:

$k_h(B) = 483\ kN/m^2$

The characteristic length of the pile structure was evaluated by employing Eq. (5.5.1) as:

$$\lambda = \sqrt[4]{\frac{200e6 * 1.355e-10}{483}} = 0.0865\ m \qquad (5.5.36)$$

The maximum length of the short pile was calculated by means of Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.0865 = 0.4325\ m \qquad (5.5.37)$$

- For cross section $B_2 = 12.7\ mm$

The moment of inertia of the model of the pile was obtained formerly in Eq. (5.4.4) as:

$I_p = 2.71e-10\ m^4$

The modulus of subgrade reaction was evaluated in Eq. (5.4.27) that gives:

$k_h(B) = 574\ kN/m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 2.71e-10}{574}} = 0.0986\ m \tag{5.5.38}$$

The maximum length of the short pile was determined by means of Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.0986 = 0.493\ m \tag{5.5.39}$$

- For cross section $B_3 = 19.05\,mm$

The moment of inertia of the model of the pile was obtained earlier in Eq. (5.4.6) as:

$I_p = 4.06e-10\ m^4$

The modulus of subgrade reaction was determined in Eq. (5.4.28), which results:

$k_h(B) = 636\ kN/m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1) as follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 4.06e-10}{636}} = 0.1067\ m \tag{5.5.40}$$

The maximum length of the short pile was determined by employing Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.1067 = 0.5335\ m \tag{5.5.41}$$

- For cross section $B_4 = 25.4\ mm$

The moment of inertia of the model of the pile was obtained formerly in Eq. (5.4.8) as:

$I_p = 5.42e-10\ m^4$

The modulus of subgrade reaction was determined in Eq. (5.4.29) that results:

$k_h(B) = 683\ kN/m^2$

The characteristic length of the pile structure was calculated by employing Eq. (5.5.1), which gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 5.42e-10}{683}} = 0.1122\ m \tag{5.5.42}$$

The maximum length of the short pile was determined by means of Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.1122 = 0.561\ m \tag{5.5.43}$$

- For cross section $B_5 = 31.75\ mm$

The moment of inertia of the model of the pile was obtained previously in Eq. (5.4.10) as:

$I_p = 6.77e-10\ m^4$

The modulus of subgrade reaction was determined in Eq. (5.4.30) as follows:

$k_h(B) = 722\ kN/m^2$

The characteristic length of the pile structure was calculated by using Eq. (5.5.1), which gives:

$$\lambda = \sqrt[4]{\frac{200e6 * 6.77e-10}{722}} = 0.117\ m \tag{5.5.44}$$

The maximum length of the short pile was evaluated by employing Eq. (5.5.3) that gives:

$$L_{MAX} = 5 * 0.117 = 0.585\ m \tag{5.5.45}$$

- For cross section $B_6 = 38.1\ mm$

The moment of inertia of the model of the pile was determined earlier in Eq. (5.4.12) as follows:

$I_p = 8.13e-10\ m^4$

The modulus of subgrade reaction was calculated in Eq. (4.4.31) as:

$k_h(B) = 756\ kN/m^2$

The characteristic length of the pile structure was obtained by employing Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 8.13e-10}{756}} = 0.1211\ m \tag{5.5.46}$$

The maximum length of the short pile was calculated by means of Eq. (5.5.3) that results:

$$L_{MAX} = 5 * 0.1211 = 0.6055\ m \tag{5.5.47}$$

- For cross section $B_7 = 44.45\ mm$

The moment of inertia of the model of the pile was obtained formerly in Eq. (5.4.14) as follows:

$I_p = 9.48e-10\ m^4$

The modulus of subgrade reaction was determined in Eq. (5.4.32) that results:

$k_h(B) = 786\ kN/m^2$

The characteristic length of the pile structure was calculated by employing Eq. (5.5.1) in accordance with:

$$\lambda = \sqrt[4]{\frac{200e6 * 9.48e-10}{786}} = 0.1246\ m \qquad (5.5.48)$$

The maximum length of the short pile was evaluated by using Eq. (5.5.3), which gives:

$$L_{MAX} = 5 * 0.1246 = 0.623\ m \qquad (5.5.49)$$

- For cross section $B_8 = 50.8\ mm$

The moment of inertia of the model of the pile was determined earlier in Eq. (5.4.16) as:

$$I_p = 10e-10\ m^4$$

The modulus of subgrade reaction was calculated in Eq. (5.4.33) as follows:

$$k_h(B) = 818\ kN/m^2$$

The characteristic length of the pile structure was evaluated by employing Eq. (5.5.1) that gives follows:

$$\lambda = \sqrt[4]{\frac{200e6 * 10e-10}{818}} = 0.125\ m \qquad (5.5.50)$$

The maximum length of the short pile was calculated by using Eq. (5.5.3) in accordance with:

$$L_{MAX} = 5 * 0.125 = 0.625\ m \qquad (5.5.51)$$

5.6. Determination of k_h for the Clayey Soil Surrounding Laterally Loaded Piles by Bowles' Method

In his review of different methods for investigation the modulus of subgrade reaction, Bowles proposed the method to determine the modulus of subgrade reaction which led to the corrected formula. This formula is independent of the properties of pile.

It depends only on the properties of soil. A correction proposed by Bowles is:

$$k_h = 24(SF)q_u + C\bar{q}N_{\bar{q}} \quad (kcf) \tag{5.6.1}$$

$$k_h = 80(SF)q_u + C\bar{q}N_{\bar{q}} \quad (kN/m^3) \tag{5.6.2}$$

where q_u is unconfined compressive strength of the soil, which it was determined in the laboratory by means of unconfined compression test.The factor $C = 24$ in Eq.(5.6.1) and $C = 80$ in Eq.(5.6.2).The $N_{\bar{q}}$ which stands for bearing capacity factor gives a depth increase, and $\bar{q}$ is effective vertical stress. SF is the factor of safety. This gives (in Fps units) for the first term $24(3)q_u = 72q_u$ for clay and is approximately two times the value of $67q_u$ suggested by Davisson and Robinson (1965).

Later Robinson (1978) found that $67s_u$ (where s_u is the cohesion) was about half the k_h indicated by a series of lateral load tests (that is, $72q_u$ was about the correct value expressed in $((kcf))$. Also the value $80(SF)q_u$ is a correct value expressed in (kN/m^3) (5.6.3)

5.6.1 Determination of the Modulus of Subgrade Reaction for Soil Type 1

The properties of soil type 1 and the model of the pile were described in Sec.5.4.1, and the unconfined shear strength $q_u = 23.94\ kPa$.

- For cross section $B_1 = 6.35\ mm$

The modulus of subgrade reaction was determined by using Eq. (5.6.3) as:

$$k_h(B) = 80 * 23.94 * 0.00635 = 12\ kPa \tag{5.6.4}$$

- For cross section $B_2 = 12.7\ mm$

The modulus of subgrade reaction was obtained by employing Eq. (5.6.3) that results:

$$k_h(B) = 80 * 23.94 * 0.0127 = 24\ kPa \tag{5.6.5}$$

- For cross section $B_3 = 19.05\,mm$

The modulus of subgrade reaction was calculated by means of Eq. (5.6.3) that results:

$$k_h(B) = 80 * 23.94 * 0.01095 = 36\ kPa \tag{5.6.6}$$

- For cross section $B_4 = 25.4\ mm$

The modulus of subgrade reaction was determined by employing Eq. (5.6.3), which gives:

$$k_h(B) = 80 * 23.94 * 0.0254 = 48\ kPa \tag{5.6.7}$$

- For cross section $B_5 = 31.75\,mm$

The modulus of subgrade reaction was evaluated by means of Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 23.94 * 0.03175 = 62\ kPa \tag{5.6.8}$$

- For cross section $B_6 = 38.1\,mm$

The modulus of subgrade reaction was calculated by using Eq. (5.6.3) that gives:

$$k_h(B) = 80 * 23.94 * 0.0381 = 74\ kPa \tag{5.6.9}$$

- For cross section $B_7 = 44.45\,mm$

The modulus of subgrade reaction was determined by means of Eq. (5.6.3) in accordance with:

$$k_h(B) = 80 * 23.94 * 0.04445 = 86\ kPa \tag{5.6.10}$$

- For cross section $B_8 = 50.8\,mm$

The modulus of subgrade reaction was obtained by using Eq. (5.6.3), which results:

$$k_h(B) = 80 * 23.94 * 0.0508 = 98\ kPa \tag{5.6.11}$$

5.6.2 Determination of the Modulus of Subgrade Reaction for Soil Type 2

The properties of soil type 2 and the model of the pile were described in sec.5.4.2, and the unconfined shear strength $q_u = 26.33\ kPa$

- For cross section $B_1 = 6.35\ mm$

The modulus of subgrade reaction was determined by using Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 26.33 * 0.00635 = 14\ kPa \tag{5.6.12}$$

- For cross section $B_2 = 12.7\ mm$

The modulus of subgrade reaction was obtained by employing Eq. (5.6.3) that gives:

$$k_h(B) = 80 * 26.33 * 0.0127 = 27\ kPa \tag{5.6.13}$$

- For cross section $B_3 = 19.05\ mm$

The modulus of subgrade reaction was calculated by means of Eq. (5.6.3), which results:

$$k_h(B) = 80 * 26.33 * 0.01095 = 40\ kPa \tag{5.6.14}$$

- For cross section $B_4 = 25.4\ mm$

The modulus of subgrade reaction was evaluated by using Eq. (5.6.3) that results:

$$k_h(B) = 80 * 26.33 * 0.0254 = 54\ kPa \tag{5.6.15}$$

- For cross section $B_5 = 31.75\ mm$

The modulus of subgrade reaction was obtained by means of Eq. (5.6.3) in accordance with:

$$k_h(B) = 80 * 26.33 * 0.03175 = 67\ kPa \tag{5.6.16}$$

- For cross section $B_6 = 38.1\ mm$

The modulus of subgrade reaction was determined by employing Eq. (5.6.3) as:

$$k_h(B) = 80 * 26.33 * 0.0381 = 80\ kPa \tag{5.6.17}$$

- For cross section $B_7 = 44.45\,mm$

The modulus of subgrade reaction was calculated by using Eq. (5.6.3) that gives:

$$k_h(B) = 80 * 26.33 * 0.04445 = 94\ kPa \tag{5.6.18}$$

- For cross section $B_8 = 50.8\,mm$

The modulus of subgrade reaction was obtained by means of Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 26.33 * 0.0508 = 107\ kPa \tag{5.6.19}$$

5.6.3 Determination of the Modulus of Subgrade Reaction for Soil Type 3

The properties of soil type 2 and the model of the pile were described in sec.5.4.2, and the unconfined shear strength $q_u = 22.5kPa$

- For cross section $B_1 = 6.35\ mm$

The modulus of subgrade reaction was determined by using Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 22.5 * 0.00635 = 11\ kPa \tag{5.6.20}$$

- For cross section $B_2 = 12.7\ mm$

The modulus of subgrade reaction was obtained by employing Eq. (5.6.3) that results:

$$k_h(B) = 80 * 22.5 * 0.0127 = 23\ kPa \tag{5.6.21}$$

- For cross section $B_3 = 19.05\,mm$

The modulus of subgrade reaction calculated by means of Eq. (5.6.3), which gives:

$$k_h(B) = 80 * 22.5 * 0.01095 = 34\ kPa \tag{5.6.22}$$

- For cross section $B_4 = 25.4\ mm$

The modulus of subgrade reaction was evaluated by using Eq. (5.6.3) in accordance with:

$$k_h(B) = 80 * 22.5 * 0.0254 = 46 \ kPa \tag{5.6.23}$$

- For cross section $B_5 = 31.75\,mm$

The modulus of subgrade reaction was calculated by employing Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 22.5 * 0.03175 = 57 \ kPa \tag{5.6.24}$$

- For cross section $B_6 = 38.1\,mm$

The modulus of subgrade reaction was obtained by means of Eq. (5.6.3) that gives:

$$k_h(B) = 80 * 22.5 * 0.0381 = 69 \ kPa \tag{5.6.25}$$

- For cross section $B_7 = 44.45\,mm$

The modulus of subgrade reaction was determined by using Eq. (5.6.3), which results:

$$k_h(B) = 80 * 22.5 * 0.04445 = 80. \ kPa \tag{5.6.26}$$

- For cross section $B_8 = 50.8\,mm$

The modulus of subgrade reaction was evaluated by employing Eq. (5.6.3) as follows:

$$k_h(B) = 80 * 22.5 * 0.0508 = 91 \ kPa \tag{5.6.27}$$

Chapter 6

Numerical Analysis Using FEM

The pile subjected to lateral loading can be analyzed using the Finite Element Method (FEM) to obtain the lateral displacement at arbitrary point by knowing the initial value of modulus of subgrade reaction, geometry of pile, and applied load.

One of the programs, which allows for verification of laboratory results by FEM is BELF. BELF is a computer program designated for the analysis of Beams on an Elastic Foundation.

The Finite Element Method is used to assemble beam's nodal equilibrium equations and an exact representation of the element displacement line is used. Thus, the results produced by the program basically does not depend on the division of the beam into elements though the mesh of elements should be adjusted to the location of applied loads and should coincide with the beam sections at which the program output values: beam deflections and internal forces are required

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6.1 The Main Features of BELF Program

a. The beam can be supported be elastic bed of a constant bed modulus of subgrade reaction within the element, which can be different in every element including the case of zero bed constant. The latter case corresponds to no support within the element. Additionally, the beam can be supported by rigid (unyielding) support constraints imposed against displacement and/or rotation and by elastic linear and rotational springs. Thus, the program can analyze beams with partial elastic foundation support and beams without any elastic bed at all.

b. The loads applied can be nodal loads (point force and point moment) and linearly distributed element loads. The program offers unlimited number of load cases. Load cases can be combined with an arbitrary multiplier assigned to each load case.

c. The program capability includes calculation of nodal displacements and element nodel forces as well as support reactions. There is practically no limitation concerning the number of elements. The program has the capability to analyze beams with as much as 300 elements, which is sufficient for all practical purposes.

d. The program is user friendly and is operated from a system of pull-down menus with a full mouse support.

e. Input in BELF prepares data to the program built in simple text editor, EDWIN, by using a problem-oriented language, as a text file. The data being edited comprises the list of elements, rigid supports, elastic supports (springs), nodal loads and distributed element loads. Simple generators obtainable as a nonstandard description of the data list row can generate Repetitive or regular pattern data. The edited data can be saved as an external text file.

f. To run a problem the user has to edit the problem data or to load a disk file with BELF input data to the program internal editor, modify it if necessary and then pull down the Solve menu and activate the Solve option. The program will check the correctness and integrity of the data solve the problem, generate an output file with the solution results and finally it will display the solution progress report.

g. If the user decides to edit a new problem data by selecting the new option from the file pull-down menu, BELF will load to the editor a file with a data template which consists of data headers and comments prompting the user with the structure of all data list and all allowable data codes.

h. The program has built in a user friendly data error diagnosis as a part of the solution procedure. In a case of an error in data, BELF emits an audible alarm and shows the insulting data line accompanied by an error message. This gives the user a chance to correct his data and with a two mouse clicks to compile the problem afresh.

Unit system:	**(kN, m)**	**(kip, inch)**
Young modulus	kN/m^2	kip/in^2
Element cross-section area	m^2	in^2
Moment of inertia	m^4	in^4
Beam subgrade modulus	kN/m^2	kip/in^2
Elastic spring constant	kN/m	kip/in
Intensity of distributed loads	kN/m	kip/in
Nodal point moment	$kN \cdot m$	$kip \cdot in$

6.2 Rules for the Data Preparation to BELF

(a) Draw a line diagram of the beam and all the loads applied to it. Divide the beam into elements by selecting the division points called nodes. Nodes should coincide with all characteristic points:

1. of the beam (abrupt change in cross section),
2. of the subgrade (abrupt change of subgrade constant) and,
3. of the applied load (section at which distributed load begins or ends or sections at which point loads are applied).

Remember to check the magnitude of the parameter lambda for the beam's average element. Nodes should be numbered sequentially starting from 1 up to Nn, when Nn is the total number of all nodes. The top node is designated as node number 1.

(b) Number elements sequentially, starting with 1 for the leftmost element. For elements supported by the elastic subgrade, check the ratio lambda = $\frac{l_e}{\lambda}$, where l_e is the element length and λ is defined by the expression $\lambda = \sqrt[4]{\frac{E_p I_p}{k_h(B)}}$, $E_p I_p$ being the element bending rigidity, $k_h(B)$ beam subgrade constant. The best results are obtained for elements of ratio lambda within the range 0.1-2.5.

All computers inputs files are written in Appendix A.

Chapter 7

Sensitivity Analysis of Laterally Loaded Piles Embedded in Homogeneous Soil

7.1 The Purpose of Sensitivity Analysis

Sensitivity analysis theory is very important to estimate the corrected certain values of design's characteristics due to the error of its initial determination. In this Chapter the sensitivity analysis of laterally loaded piles is presented. The numerical analysis gives a certain displacement for the model of the pile due to lateral load at the end of the pile employing the initial value of modulus of subgrade reaction. By comparison these values by that obtained from the experimental test for the same model of the pile, the difference in both results was observed. By knowing the first variations of displacement, approximate values of first variation of modulus of subgrade reaction, which added to the initial values was determined to evaluate the corrected value of modulus of subgrade reaction.

A simple one-dimensional idealisation in conjunction with the beam on elastic foundation approach is used for sensitivity analysis of laterally loaded piles in soil. The first variation of arbitrary displacement and an internal force at a specific cross section due to some variations of design variable are derived using the adjoint method. The pile cross section dimensions, the pile material and the soil parameters are considered to be the design variables.

The identification of the correct value of modulus of subgrade reaction is performed in framework of sensitivity theory.

Consider a pile embedded in a homogenous soil as shown in Fig. 7.1. The pile is subjected to lateral load F. The pile structure is modelled as a beam element with the bending stiffness EI. It is analysed as a beam on elastic foundation of Winkler type.

In this way defined structure is called the primary structure. The behaviour of that structure is analysed in co-ordinate system z, v as shown in Fig.7.1.The variable z defines spatial variable, while v denotes variable lateral displacement. This means, that the following constitutive relationships are satisfied.

1. for the pile subject to bending:

$$M = -EIv'' \tag{7.1}$$

2. for soil reaction.

$$p = k_h(B)v \tag{7.2}$$

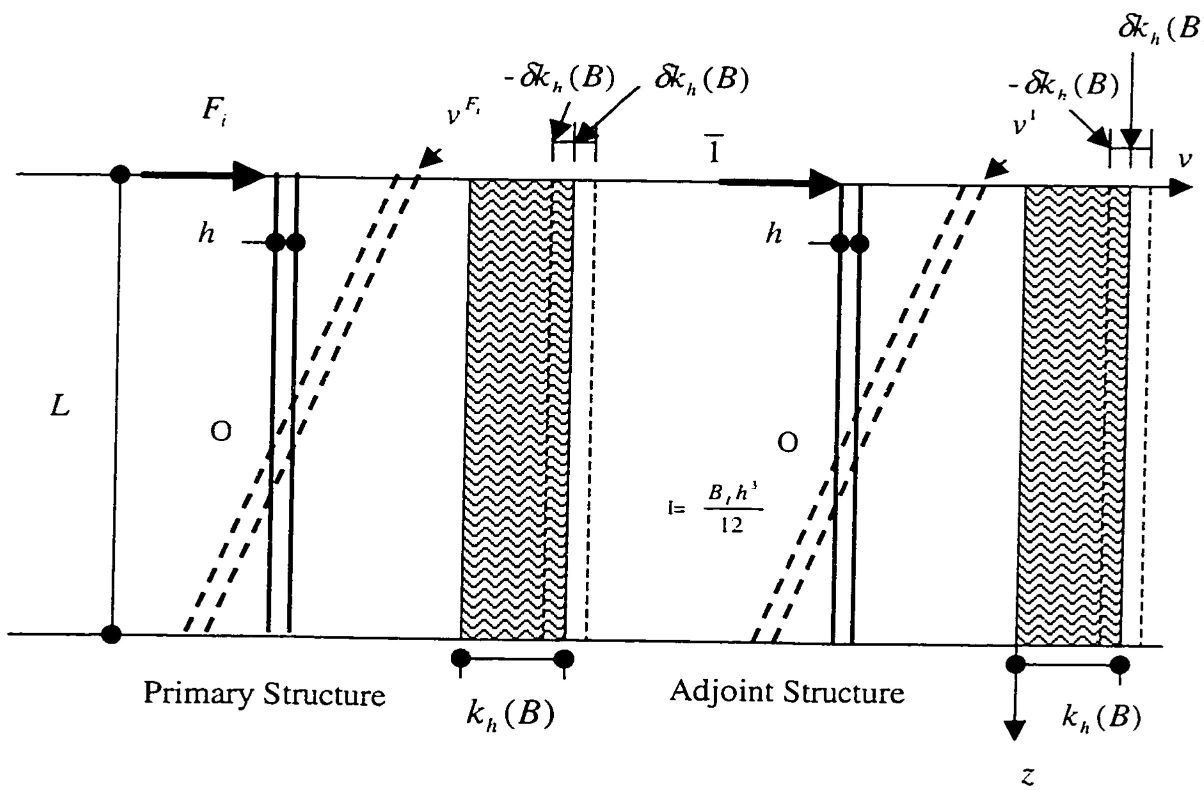

Figure 7.1 The Geometry, Load Conditions Displacement Fields and Variability of Modulus of Subgrade Reaction $k_h(B)$ for Primary and Adjoint Structure for Pile in Cohesive Soil used in Sensitivity Analysis.

The change of M and p can be expressed in terms of only state variables v and v'' as:

$$\delta M = -EI\delta v'' \tag{7.3}$$

$$\delta p = k_h(B)\delta v \tag{7.4}$$

Some statically admissible variation of displacement field is imposed on the original structure, which is subject to constant load.

For the given state, the change of lateral displacement at arbitrary point can be determined by means of virtual work theorem with aid of adjoint structure concept, which is made of the same material, has the same dimensions and boundary conditions as the primary structure and corresponds to the state of determination of the primary structure. The adjoint structure is subject to unit load $\overline{1}$ applied at the upper end of the pile. Thus:

$$\overline{1} \cdot \delta v = \int_0^l (-\overline{M}\delta v'' + \overline{p}\delta v) \cdot dz \tag{7.5}$$

where:

$\overline{M}, \overline{p}$ are internal bending moment and soil reaction of the adjoint structure obtained as the results of application unit load $\overline{1}$ at the upper end of the pile.

$\delta v'', \delta v$ are the variations of the generalized displacement imposed on the primary structure, which is subjected to constant load conditions.

The variation δv on the left-hand side of Eq. (7.5) is caused due to the changes (variations) of variables $\delta(EI)$ and $\delta(k_h(B))$ which are considered as the design variable. The Eqs. (7.3) and (7.4)can be written in the scope of sensitivity theory as:

$$\delta M = -\delta(EI)v'' - EI\delta v'' \tag{7.6}$$

$$\delta p = \delta(k_h(B))v + k_h(B)\delta v \tag{7.7}$$

However, the statically admissible variation of displacement field is imposed on the primary structure are in the presence of constant load conditions. This means, that variations of the internal forces of primary structure described by Eqs. (7.6) and (7.7) must vanish.

Consequently, based on the above-described condition, the variations of $\delta v''$ and δv required by Eq. (7.5) can be determined from Eqs. (7.6) and (7.7). Thus,

$$\delta v'' = -\frac{\delta(EI)v''}{EI} \tag{7.8}$$

$$\delta v = -\frac{\delta(k_h(B))v}{k_h(B)} \tag{7.9}$$

Substituting Eqs. (7.8) and (7.9) into Eq. (7.5), gives:

$$\overline{1} \cdot \delta v = \int_0^l [\frac{\overline{M} \cdot v''}{EI} \cdot \delta(EI) - \overline{p} \cdot \frac{v}{k_h(B)} \cdot \delta(K_h(B))] \cdot dz \tag{7.10}$$

Employing the constitutive relationships (7.1) and (7.2) in Eq. (6.10), the δv can be expressed as follows:

$$\overline{1} \cdot \delta v = \int_0^l \left[-\overline{v}'' \cdot v'' \cdot \delta(EI) - \overline{v} \cdot v \cdot \delta(k_h(B)) \right] \cdot dz \tag{7.11}$$

where:

$\overline{v}'', \overline{v}$ represent the generalized lateral displacement of the adjoint structure distributed along the pile axis.

v'', v denote the generalized lateral displacement of the primary structure subject to given load conditions.

The above equation represents the sensitivity of lateral displacement v due to the changes of bending stiffness of the pile $\delta(EI)$ and the modulus of subgrade reaction $\delta(k_h(B))$.

Eq. (7.11) can used for identification of the value of EI or $k_h(B)$ if the δv is known. In the investigated research δv is defined as error between laboratory recorded displacement and the outcome of numerical analysis evaluated by BELF, which is dependent on the value of $k_h(B)$. Thus the error of the displacement δv will vanish if the $k_h(B)$ will be corrected by the value of $\delta(k_h(B))$.

Thus assuming that the cross section of the model of the piles is constant ($\delta(EI) = 0$), then the Eq. (7.11) is reduced to the following:

$$\overline{1} \cdot \delta v = -\int_0^l \overline{v} \cdot v \cdot \delta(k_h(B)) \cdot dz \tag{7.12}$$

It is correct for given homogenous soil. Equation (7.12), which is now used for identification of correct value of, $k_h(B)$ can be written as:

$$\delta(k_h(B)) = -\frac{\delta v}{\int_0^L \overline{v} \cdot v \cdot dz} \tag{7.13}$$

where:

v is the displacement at any arbitrary point of the primary structure.

It can be determined from the linear deflection equation of the primary structure as follows:

$$v = \frac{v_{Top} - v_{bot}}{L} \cdot z + v_{Top} \tag{7.14}$$

$\bar{v}$ is the displacement at any arbitrary point of the adjoint structure.

It can be obtained from the linear deflection equation of the adjoint structure as follows:

$$\bar{v} = \frac{\bar{v}_{Top} - \bar{v}_{bot}}{L} \cdot z + \bar{v}_{Top} \tag{7.15}$$

where:

v_{Top} stands for the lateral displacement at the upper end of the pile for primary structure,

v_{bot} stands for the lateral displacement at the lower end of the pile for primary structure,

$\bar{v}_{Top}$ means the lateral displacement at the upper end of the pile for adjoint structure,

$\bar{v}_{bot}$ denotes the lateral displacement at the lower end of the pile for adjoint structure.

The integration in the denominator of the Eq. (7.13) is made by means of Simpson's method.

It is worth to remind the meaning of v and $\bar{v}$ in the Eq. (7.13). They represent lateral displacement in the primary and adjoint structure respectively distributed along the pile axis.These function have to be determined for analyzed load conditions either analytically or numerically.

7.2 The Difference Between Sensitivity Formulation and Identification Process

Equation. (7.13) obtained as special case of sensitivity of lateral displacement v at arbitrary cross section due to the change of modulus of subgrade reaction k_h (B) can be employed in identification of true value of material characteristics of the soil medium in which the pile structure is embedded. The typical approach of physical or material identification problem is as follows:

The mathematical model, which defines the behaviour of the system, is given. This model requires to its description some physical parameters, which can be determined experimentally based on experimental methods. The reliability of experimental method means, that physical parameters determined in laboratory should give the same results as well as those obtained from theoretical analysis. If the discrepancy between the theoretical and experimental results exists, it is postulated that is associated with incorrectly determined parameters.

The lateral displacement v_{Lab} at the upper end of the pile was obtained due to the lateral force acted on the upper end in the laboratory.

Based on the method of application for determination the modulus of subgrade reaction and the material properties $k_h(B)$ was obtained in the laboratory. Bowles' and Vesic's method were used to determine the modules of subgrade reaction for clayey soil and Terzaghi's method was used to determine the modules of subgrade reaction for both clayey and sandy soil as described previously in Chapter 4.

Using the numerical analysis (Finite Element Method) to analyse the model of the pile using and obtain the lateral displacement at the top of the pile by employing the previous modulus of subgrade reaction obtained in Chapter 4. And by knowing the dimensions, properties of the models of the piles and the applied load, the corresponding lateral displacement at the upper end of the model of the pile was determined $v_{T(B)}$. The difference between the lateral displacement obtained from the laboratory experimental v_{Lab} and that obtained from numerical analysis $v_{T(B)}$ (if there difference) for the same load condition and the same geometry allows us to determine the variation of the lateral displacement at the upper end of the pile.

$$\delta v = v_{Lab} - v_{T(B)} \tag{7.16}$$

This difference represents the error, which is postulated to be associated with incorrectly determined physical parameter $k_h(B)$.

It is worth noting, that in the Eq. (7.11) the distribution of $\delta(EI)$ and $\delta(k_h(B))$ are considered known and the unknown is δv. In the identification problem in Eq. (7.16) the δv is given and the unknown is $\delta(k_h(B))$, which can be obtained from Eq (7.13). The sought correction of physical parameter assures the minimization of the error of deformation between experimental and theoretical model.

7.3 Application of Sensitivity Analysis in the Determination of $\delta(k_h(B))$ for Clayey Soil

Due the fact that $k_h(B)$ is constant along the length of the pile, so the variation of the modulus of subgrade reaction $\delta(k_h(B))$ is also constant along the length of the pile. Thus $\delta(k_h(B))$ can be obtained from Eq. (7.13) as described previously.

$$\delta(k_h(B)) = -\frac{\delta v}{\int_0^L \bar{v} \cdot v \cdot dz} \tag{7.17}$$

7.4 Application of Sensitivity Analysis in the Determination of δn_h for Sandy Soil

By using the same Eq.(7.13), δn_h can obtained taking in consideration that $k_h(B)$ for sandy soil is variable along the length of the pile. It represented by linear variability along the depth as shown in Fig. 7.2

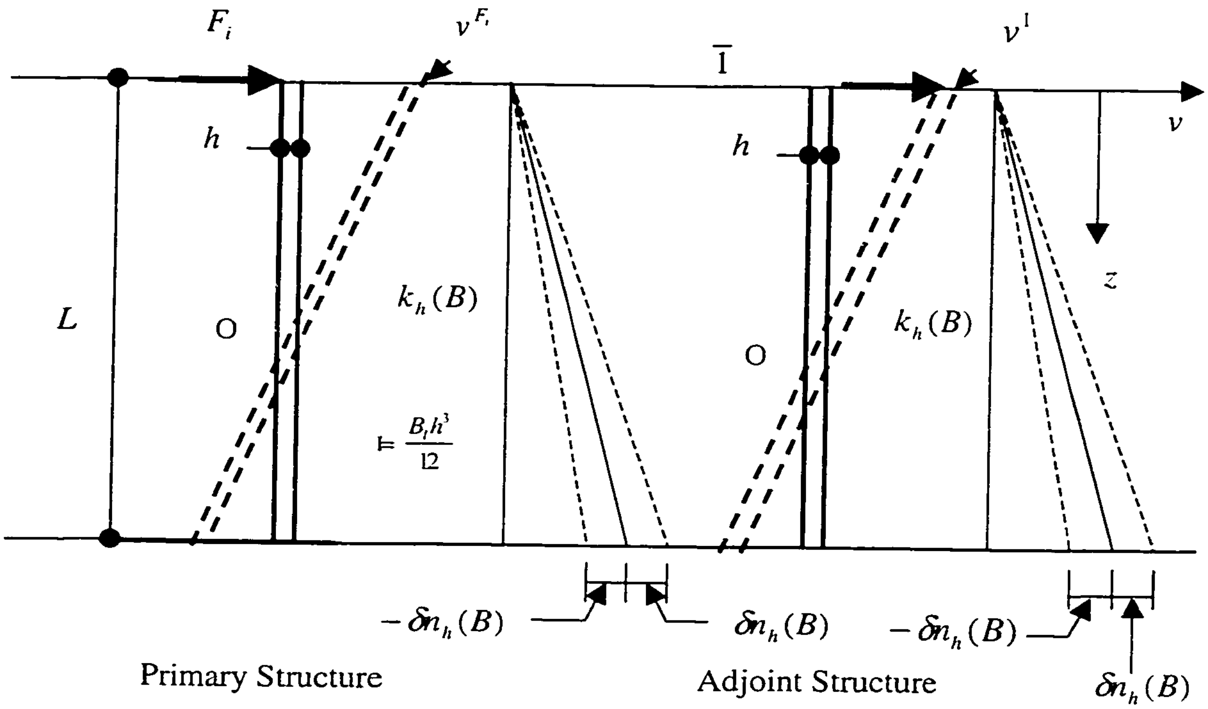

Figure 7.2 The Geometry, Load Conditions Displacement Fields and Variability of Modulus of Subgrade Reaction $k_h(B)$ for Primary and Adjoint Structure for Pile in Frictional Soil used in Sensitivity Analysis.

It is important to know that $n_h(B)$ is constant.

The following notations are used in identification of $n_h(B)$

v represents the distribution of lateral displacement along the pile length produced by lateral force F in laboratory.

$\bar{v}$ represents the distribution of lateral displacement along the pile length produced by unit load $\overline{1}$ applied at the upper end of the pile of the adjoint structure.

The sensitivity equation given by Equation (7.13), which will be used in identification of, $n_h(B)$ requires some modification. This connected with variation of $\delta n_h(B)$. The modulus of subgrade reaction was obtained from Chapter 4 in Eq. (4.3.3).

$$k_h(B) = n_h(B) \cdot z \tag{4.3.3}$$

The variation of $n_h(B)$ is defined as:

$$\delta k_h(B) = \delta n_h(B) \cdot z \tag{7.18}$$

where:

$\delta n_h(B)$ is the variation of $n_h(B)$ which minimises the error of displacement δv.

z stands for the arbitrary depth of embedment of the pile.

Substitution by Eq. (7.18) into Eq. (7.13) results:

$$\delta n_h(B) \cdot z = -\frac{\delta v}{\int_0^L \bar{v} \cdot v \cdot dz} \tag{7.19}$$

Since the $\delta n_h(B)$ is constant value, therefore it can be taken in front of the integral. Thus:

$$\delta n_h(B) = -\frac{\delta v}{\int_0^L \bar{v} \cdot v \cdot z \cdot dz} \tag{7.20}$$

7.5 Identification of Modulus of Subgrade Reaction for all Discrete Points of a Performance Curves

The variability of $k(B)$ is investigated such to express the dependency on B_i explicitly as the product of [$k_{h(i)} \times (B_i)$].

This way allows also for independent evaluation of $k_{h(i)}$ associated with each load's application for each performance curve, which is determined in the framework of sensitivity theory employing the state variable method, which is also known as the conjugated system method.

The assessment of new values of $[k_{h(i)} \times (B_i)]_n$ or $[k_{h(i)}]_n$ is conducted with respect to the appropriate initial value of $k_{h(i)} \times (B_i)$ or $k_{h(i)}$.

The approximation of the performance curves by a straight line, for each application of the force produces difference of the displacement δv, which is caused, by the difference in modulus of subgrade reaction $\delta k_{h(i)}$ as described previously in this Chapter.

The variability of δk_{h1} and δk_{h8} associated with B_1 and B_8 respectively for laboratory determination performance curves are shown in Fig. 7.3.

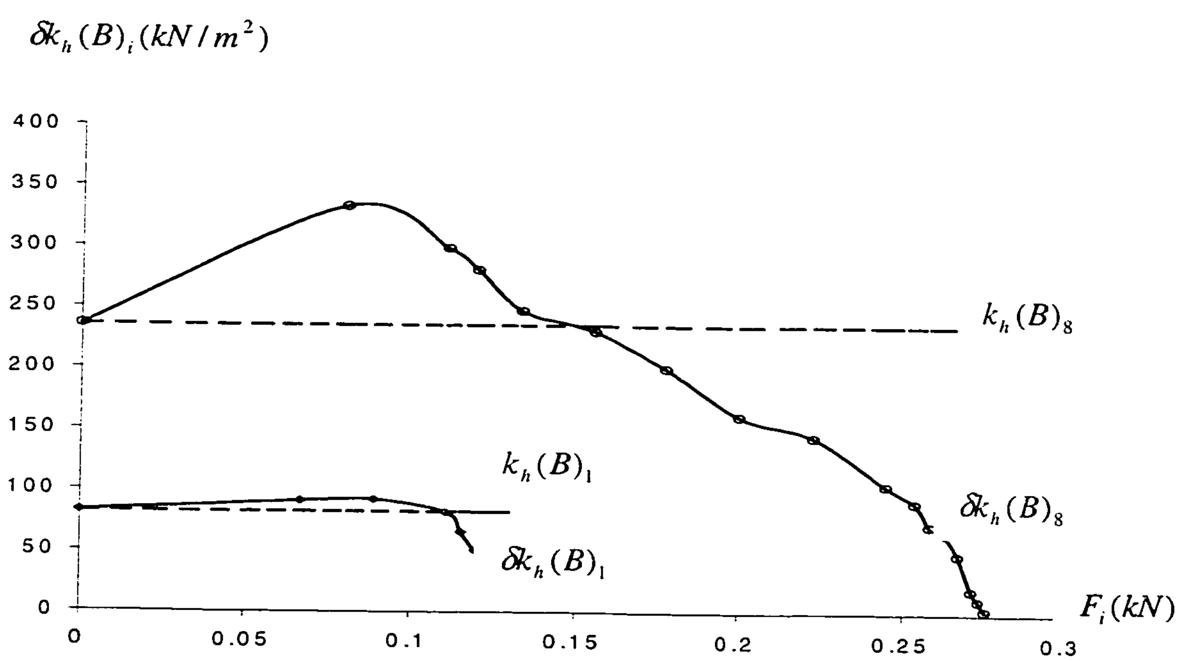

Figure 7.3 Distribution of δk_{h1} and δk_{h8} for Discrete Force Points F_i.

The examination if the modulus of subgrade reaction used in analysis of piles is width dependent is conducted with assistance of uniform line load q distributed across the width B_i of the model of pile, then the concentrated force F_i is linearly proportional to q, since it is product of q and B_i. On the other hand, the moment of inertia of the model of the pile is also linearly dependent on B_i. For such a case the deflection of a beam on Winkler foundation (surrounding by air) is independent of B_i and proportional to q.

The approach described is confronted with laboratory results, which are shown in Figs. 7.4 and 7.5. The former presents the displacement v of control point for variable B_i and constant value of $q = 3.5\ kN/m$ while latter demonstrates the variability of $k_h(B)$ associated with different B_i for constant $q = 5\ kN/m$.

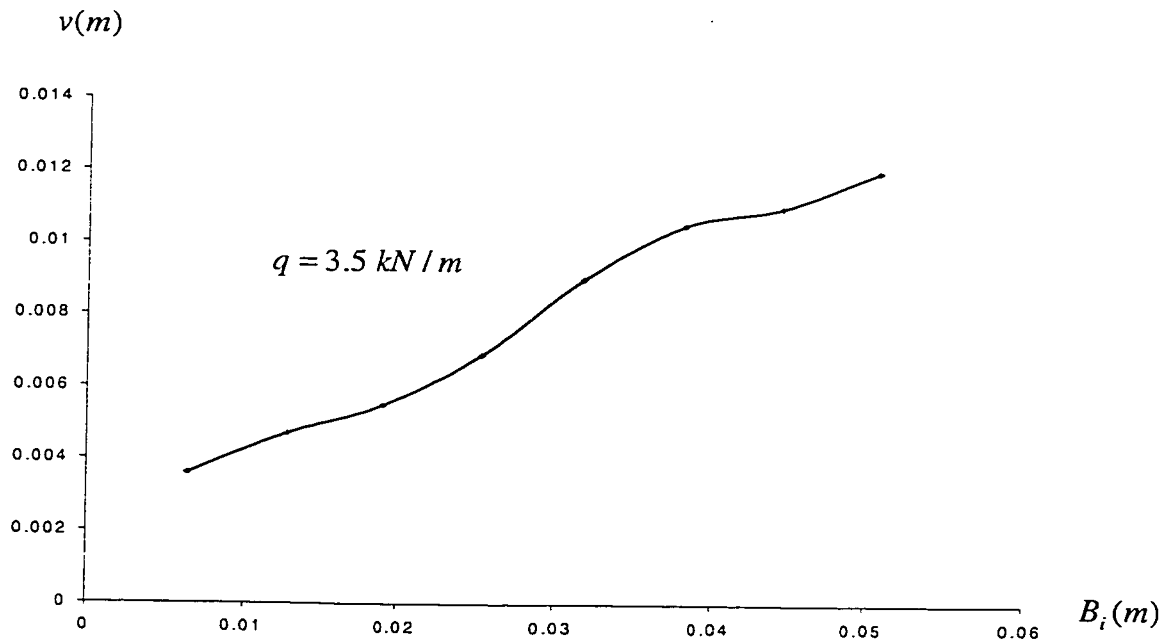

Figure 7.4 Variability of the Displacement v of Control Point for q=3.5 kN/m and Variable B_i.

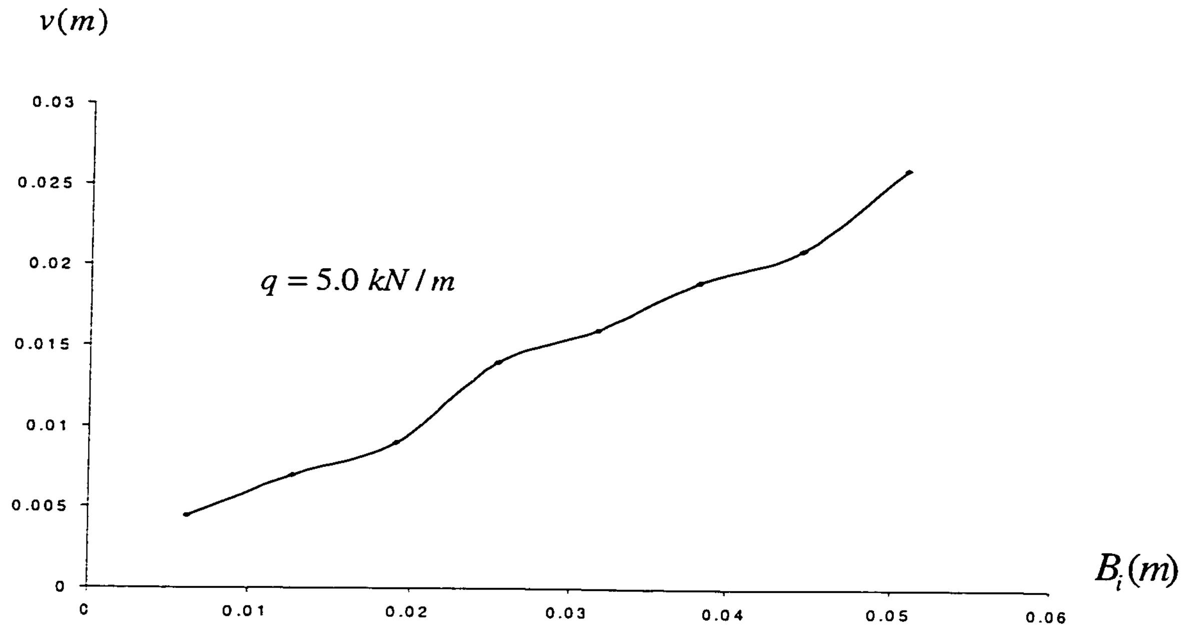

Figure 7.5 Variability of the Displacement v of Control Point for q =5.0 kN/m and Variable B_i.

The curves for all values of q for sandy and clayey soil are in Appendix C.

Also the curves show the relationships between the width of the piles and the modulus of subgrade reaction for piles embedded in sandy and clayey soil are in Appendix C.

Chapter 8

Analysis of Experimental Results and Discussion on Identification Investigation

8.1 Discussion on Parameter Associated with Soil Behavior Surrounding Laterally Loaded Piles

The survey of literature on laterally loaded piles generated some questions on irregularities in definitions of the constant of modulus of subgrade reaction $n_h(B)$ and the modulus of subgrade reaction $k_h(B)$. The first one is connected with linear variability of modulus of subgrade reaction and is used in simulation of the sandy soil by means of system of springs, while the second parameter $k_h(B)$ with its constant value is associated with modeling of clayey soil by means of system of springs. This important soil parameter attracted the attention of Terzaghi who wrote a paper to clarify that problem. It is connected with erroneous interpretation of the meaning of the modulus of subgrade reaction, which is used in analysis of beams on elastic foundation as described briefly in Chapter 4.

It is demonstrated by the following differential equation of the problem employed in analysis of laterally loaded piles:

$$EI\frac{d^4v}{dz^4}+k_h\cdot B\cdot v=0 \qquad (4.1.7)$$

The question, which appears is connected with the second term of Eq. (4.1.7). That is if the Eq. (4.1.7) is correct or should be rather written for the purpose of analysis of laterally loaded piles as:

$$EI\frac{d^4 v}{dz^4}+k_h(B)v=0 \tag{4.1.8}$$

Laboratory determined numerical values of $k_h(B)$ and $n_h(B)$ have been used for verification in BELF program which requires values of modulus of subgrade reaction associated with width of the pile. All computer programs, which were written to obtain the displacement at the top of the pile, are presented in Appendix A. The obtained numerical results were compared with laboratory results. The discrepancy between the laboratory results and numerical investigation is associated with approximate values of $k_h(B)$ and $n_h(B)$ determined based on Terzaghi's, Bowles', and Vesic's approach. Some of these formulas (Vesic's method) take into account width as well as pile's stiffness while other (Bowles' method) is independent of width. The approximated assessment of k by means of various formulas often results in considerable irregularities.

The initial value of modulus of subgrade reaction obtained by Terzaghi was represent by straight line, which interpolates the force-displacement curve from laboratory tests. It is worth noting that the values of modulus of subgrade reaction obtained from both Vesic's and Bowles are completely different that obtain from Terzaghi. The method of sensitivity analysis did not give the correct value of modulus of subgrade reaction for that obtained from both Vesic and Bowles. On the other hand it gave exact value of variation of modulus of subgrade reaction which added to the initial value of modulus of subgrade reaction .The same value of lateral displacement was obtained theoretically through BELF by using this correct value.

8.2 Characteristic Features of Experimental Results and Identification Process of Laterally Loaded Piles

The laboratory investigations were performed for short piles, the length of which is given by the relationship.

$$L \leq 5\lambda \tag{4.5.3}$$

where:

λ stands for characteristics length of the pile.

When $L \leq 5\lambda$, laterally loaded pile behaves as rigid pile and its deformation is obtained through rotation about point o_1 located at a certain depth, and it is considered as short pile.

The laboratory results in terms of tables and performance curves for a short model of the pile subjected to variable horizontal forces at the top of model of the pile are presented in Appendix B. Also the value of modulus of subgrade reaction and the corrected value from sensitivity analysis are presented in Appendix B. Moreover, all curves which represent the relationship between the load, displacement and modulus of subgrade reaction are presented in Appendix C.

Chapter 9

Summary and Conclusion

The research is focused on investigations of width dependent modulus of subgrade reaction when used in analysis of laterally loaded piles. The relationship between the applied loads and displacements of a control point for models of the piles having variable width are determined in laboratory. The models of the piles are analyzed by static method to determine first approximation of modulus of subgrade reaction. Then they are adjusted to accurate values in the framework of sensitivity theory employing state space method known also as adjoint structure method. It is done by means of application of dummy load to the adjoint structure. The functional of energy is determined in the scope of variational calculus. The equation of first variation of lateral displacement of control point due to variation of modulus of subgrade reaction forms basis for determination of sought changes of modulus of subgrade reaction, which guarantee the concurrence of approximated results with those determined in laboratory.

The assessment of modulus of subgrade reaction is established with involvement of uniform line load q applied across the width of the models of the piles..

The first stage for detection of inaccurate initial value of modulus of subgrade reaction is based on verification of laboratory results by means of FEM employing the initially determined values of modulus of subgrade reaction obtained using Terzaghi's, Bowles', and Vesic's method. Big difference for all three values of initial modulus of subgrade reaction was observed. The errors of displacement of control point form basis for determination of satisfactory values of modulus of subgrade reaction for clay or constant of horizontal subgrade reaction of sand employing the sensitivity analysis theory.

The following conclusions were observed:

1. The modulus of subgrade reaction used in analysis of laterally loaded piles is width dependent.
2. The correct values of modulus of subgrade reaction have been obtained by incorporation of sensitivity analysis with Terzaghi's method.
3. Bowles' and Vesic's method did not give the correct values of modulus of subgrade reaction even employing the sensitivity analysis.

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Appendix A

COMPUTER INPUTS (BELF)

Computer Inputs (BELF)

A.1 Computer Program for a Pile Embedded in Clayey Soil by Terzaghi's Method for Determination of k.

A.1.1 For Pile Width = 6.35 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L          Iy          k
  1         0.02      1.355E-10      0
  2         0.02      1.355E-10    82.514
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}
Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y   0.044482
Local
* El       V1    (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.1.2 For Pile Width = 1.27 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L        Iy         k
  1        0.02     2.71e-10     0
  2        0.02     2.71e-10   103.138
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1     Y    0.044482
Local
* El     V1     (V2)  {or Ej to Ek  V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.1.3 For Pile Width = 19.05 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L        Iy          k
  1        0.02     4.06e-10      0
  2        0.02     4.06e-10    145.62
 20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0.044482
Local
* El      V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    .. Next load cases and/or Combine directives
Stop
```

A.1.4 For Pile Width = 25.4 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L         Iy          k
  1        0.02     5.42e-10       0
  2        0.02     5.42e-10    154.707
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y   0.044482
Local
* El      V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.1.5 For Pile Width = 31.75 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected toLateral Force = 0.044482 kN**.**

```
Elements=20
* El.        L         Iy          k
  1        0.02     6.77e-10       0
  2        0.02     6.77e-10    157.17
  20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y   0.044482
Local
* El      V1    (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.1.6 For Pile Width = 38.1 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L        Iy          k
  1        0.02     8.13e-10      0
  2        0.02     8.13e-10    165.028
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.1.7 For Pile Width = 44.45 *mm*

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 kN**.**

```
Elements=20
* El.        L         Iy          k
  1         0.02      9.48e-10     0
  2         0.02      9.48e-10   220.04
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y     0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.1.8 For Pile Width = 50.8 mm

Title: Analysis of a Short Pile Embedded in Clayey Soil and is Subjected to Lateral Force = 0.044482 kN**.**

```
Elements=20
* El.        L         Iy        k
  1        0.02      10e-10      0
  2        0.02      10e-10    235.754
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y   0 .044482
Local
* El     V1      (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.2 Computer Program for a Pile Embedded in Sandy Soil by Terzaghi's Method for Determination of n_h

A.2.1 For Pile Width = 6.35 mm

Title: Analysis of a Short pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L         Iy          k
  1        .0175     1.355e-10     0
  2        .0175     1.355e-10     4.2867
  3        .0175     1.355e-10     12.86
  4        .0175     1.355e-10     21.4338
  5        .0175     1.355e-10     30
  6        .0175     1.355e-10     38.581
  7        .0175     1.355e-10     47.1544
  8        .0175     1.355e-10     55.728
  9        .0175     1.355e-10     64.3
  10       .0175     1.355e-10     72.875
  11       .0175     1.355e-10     81.448
  12       .0175     1.355e-10     90.02
  13       .0175     1.355e-10     98.596
  14       .0175     1.355e-10     107.169
  15       .0175     1.355e-10     115.743
  16       .0175     1.355e-10     124.3162
  17       .0175     1.355e-10     132.889
  18       .0175     1.355e-10     141.4632
  19       .0175     1.355e-10     150.04
  20       .0175     1.355e-10     158.61
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1     Y     .044482
Local
* El    V1    (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.2 For Pile Width = 12.7 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L          Iy          k
  1        .0175      2.71e-10     0
  2        .0175      2.71e-10    4.626
  3        .0175      2.71e-10    13.878
  4        .0175      2.71e-10    23.13
  5        .0175      2.71e-10    32.382
  6        .0175      2.71e-10    41.634
  7        .0175      2.71e-10    50.886
  8        .0175      2.71e-10    60.138
  9        .0175      2.71e-10    69.39
  10       .0175      2.71e-10    78.642
  11       .0175      2.71e-10    87.89
  12       .0175      2.71e-10    97.146
  13       .0175      2.71e-10    106.398
  14       .0175      2.71e-10    116.65
  15       .0175      2.71e-10    124.9
  16       .0175      2.71e-10    134.154
  17       .0175      2.71e-10    143.41
  18       .0175      2.71e-10    152.658
  19       .0175      2.71e-10    161.91
  20       .0175      2.71e-10    171.162
E=200e6
Supports=0
* N  code       {code=Y,FIX,SY}
Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    .044482
Local
* El    V1     (V2)  {or Ej to Ek  V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.3 For Pile Width = 19.05 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.      L        Iy         k
 1       .0175    4.06e-10    0
 2       .0175    4.06e-10    6.438
 3       .0175    4.06e-10    19.313
 4       .0175    4.06e-10    32.189
 5       .0175    4.06e-10    45.06
 6       .0175    4.06e-10    57.941
 7       .0175    4.06e-10    70.816
 8       .0175    4.06e-10    83.692
 9       .0175    4.06e-10    96.568
 10      .0175    4.06e-10    109.44
 11      .0175    4.06e-10    122.319
 12      .0175    4.06e-10    135.195
 13      .0175    4.06e-10    148.07
 14      .0175    4.06e-10    160.946
 15      .0175    4.06e-10    173.822
 16      .0175    4.06e-10    186.697
 17      .0175    4.06e-10    199.573
 18      .0175    4.06e-10    212.45
 19      .0175    4.06e-10    225.325
 20      .0175    4.06e-10    238.2
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y     .044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.4 For Pile Width = 25.4 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L          Iy          k
  1        .0175      5.42e-10     0
  2        .0175      5.42e-10     6.457
  3        .0175      5.42e-10     19.371
  4        .0175      5.42e-10     32.286
  5        .0175      5.42e-10     45.2
  6        .0175      5.42e-10     58.114
  7        .0175      5.42e-10     71.03
  8        .0175      5.42e-10     83.943
  9        .0175      5.42e-10     96.857
  10       .0175      5.42e-10     109.77
  11       .0175      5.42e-10     122.686
  12       .0175      5.42e-10     135.6
  13       .0175      5.42e-10     148.514
  14       .0175      5.42e-10     161.428
  15       .0175      5.42e-10     174.343
  16       .0175      5.42e-10     187.257
  17       .0175      5.42e-10     200.172
  18       .0175      5.42e-10     213.086
  19       .0175      5.42e-10     226
  20       .0175      5.42e-10     238.914
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
 1       Y   .044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.5 For Pile Width = 31.75 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

Elements=20

* El.	L	Iy	*k*
1	.0175	6.77e-10	0
2	.0175	6.77e-10	10.717
3	.0175	6.77e-10	32.151
4	.0175	6.77e-10	53.5845
5	.0175	6.77e-10	75.02
6	.0175	6.77e-10	96.4522
7	.0175	6.77e-10	117.89
8	.0175	6.77e-10	139.32
9	.0175	6.77e-10	160.753
10	.0175	6.77e-10	182.187
11	.0175	6.77e-10	203.62
12	.0175	6.77e-10	225.055
13	.0175	6.77e-10	246.49
14	.0175	6.77e-10	267.923
15	.0175	6.77e-10	289.36
16	.0175	6.77e-10	310.79
17	.0175	6.77e-10	332.224
18	.0175	6.77e-10	353.66
19	.0175	6.77e-10	375.1
20	.0175	6.77e-10	396.526

```
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
 1       Y   .044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.6 For Pile Width = 38.1 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.       L         Iy          k
  1       .0175     8.13e-10     0
  2       .0175     8.13e-10    15.42
  3       .0175     8.13e-10    46.26
  4       .0175     8.13e-10    77.1
  5       .0175     8.13e-10    107.94
  6       .0175     8.13e-10    138.78
  7       .0175     8.13e-10    169.62
  8       .0175     8.13e-10    200.46
  9       .0175     8.13e-10    231.3
  10      .0175     8.13e-10    262.14
  11      .0175     8.13e-10    292.98
  12      .0175     8.13e-10    323.82
  13      .0175     8.13e-10    354.66
  14      .0175     8.13e-10    385.5
  15      .0175     8.13e-10    416.34
  16      .0175     8.13e-10    447.181
  17      .0175     8.13e-10    478.021
  18      .0175     8.13e-10    508.86
  19      .0175     8.13e-10    539.7
  20      .0175     8.13e-10    570.541
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
 1       Y   .044482
Local
* El      V1   (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.7 For Pile Width = 44.45 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L          Iy          k
  1        .0175      9.48e-10     0
  2        .0175      9.48e-10     17.1477
  3        .0175      9.48e-10     51.4433
  4        .0175      9.48e-10     85.7388
  5        .0175      9.48e-10     120.034
  6        .0175      9.48e-10     154.33
  7        .0175      9.48e-10     188.625
  8        .0175      9.48e-10     222.92
  9        .0175      9.48e-10     257.216
  10       .0175      9.48e-10     291.512
  11       .0175      9.48e-10     325.807
  12       .0175      9.48e-10     360.103
  13       .0175      9.48e-10     394.4
  14       .0175      9.48e-10     428.694
  15       .0175      9.48e-10     462.99
  16       .0175      9.48e-10     497.285
  17       .0175      9.48e-10     531.581
  18       .0175      9.48e-10     565.876
  19       .0175      9.48e-10     600.172
  20       .0175
Supports=0
* N  code      {code=Y,FIX,SY}
Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
 1       Y    .044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)}
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.2.8 For Pile Width = 50.8 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.      L        Iy        k
  1      .0175    10e-10    0
  2      .0175    10e-10    18.863
  3      .0175    10e-10    56.587
  4      .0175    10e-10    94.313
  5      .0175    10e-10    132.038
  6      .0175    10e-10    169.763
  7      .0175    10e-10    207.488
  8      .0175    10e-10    245.213
  9      .0175    10e-10    282.94
  10     .0175    10e-10    320.66
  11     .0175    10e-10    358.39
  12     .0175    10e-10    396.114
  13     .0175    10e-10    433.838
  14     .0175    10e-10    471.564
  15     .0175    10e-10    509.289
  16     .0175    10e-10    547.014
  17     .0175    10e-10    584.74
  18     .0175    10e-10    622.45
  19     .0175    10e-10    660.19
  20     .0175    10e-10    697.914
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}
Springs=0
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
Load Case: #1
Global
* N     dir     V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
 1       Y   .044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)
Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.3 Computer Program for a Pile Embedded in Clayey Soil by Bowles' Method for Determination of k

A.3.1 For Pile Width = 6.35 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.          L          Iy         k
  1           0.02      1.355E-10    0
  2           0.02      1.355E-10   12
  20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    0.044482
Local
* El      V1    (V2)  {or Ej to Ek  V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.3.2 For Pile Width = 12.7 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L         Iy          k
  1        0.02     2.71e-10      0
  2        0.02     2.71e-10      24
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    0.044482
Local
* El       V1      (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
     ... Next load cases and/or Combine directives
Stop
```

A.3.3 For Pile Width = 19.05 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN* **.**

```
Elements=20
* El.        L        Iy         k
  1         0.02     4.06e-10    0
  2         0.02     4.06e-10    36
 20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0.044482
Local
* El     V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
     .. Next load cases and/or Combine directives
Stop
```

A.3.4 For Pile Width = 25.4 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.          L          Iy          k
  1          0.02      5.42e-10       0
  2          0.02      5.42e-10      48
  20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1        Y   0.044482
Local
* El      V1    (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.3.5 For Pile Width = 31.75 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.          L          Iy          k
  1          0.02       6.77e-10      0
  2          0.02       6.77e-10     60
 20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    0.044482
Local
* El      V1    (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.3.6 For Pile Width = 38.1 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.       L        Iy        k
  1        0.02    8.13e-10    0
  2        0.02    8.13e-10   72
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N     dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.3.7 For Pile Width = 44.45 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN**.**

```
Elements=20
* El.         L         Iy          k
 1           0.02      9.48e-10    0
 2           0.02      9.48e-10    84
 20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y     0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.3.8 For Pile Width = 50.8 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.          L          Iy          k
 1           0.02       10e-10        0
 2           0.02       10e-10       96
 20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    0 .044482
Local
* El     V1      (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
   ... Next load cases and/or Combine directives
Stop
```

A.4 Computer Program for a Pile Embedded in Clayey Soil by Vesic's Method for Determination of k

A.4.1 For Pile Width = 6.35 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L        Iy          k
  1         0.02     1.355E-10    0
  2         0.02     1.355E-10   429
  20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y   0.044482
Local
* El      V1    (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
* ... Next load cases and/or Combine directives
Stop
```

A.4.2 For Pile Width = 12.7 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.          L          Iy          k
  1          0.02       2.71e-10     0
  2          0.02       2.71e-10    510
 20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir        V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir        V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y     0.044482
Local
* El       V1       (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.4.3 For Pile Width = 19.05 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.         L         Iy         k
  1         0.02     4.06e-10      0
  2         0.02     4.06e-10    565
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y     0.044482
Local
* El      V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    .. Next load cases and/or Combine directives
Stop
```

A.4.4 For Pile Width = 25.4 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.          L          Iy           k
  1          0.02      5.42e-10        0
  2          0.02      5.42e-10      607
 20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir       V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1        Y   0.044482
Local
* El       V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.4.5 For Pile Width = 31.75 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN **.**

```
Elements=20
* El.          L          Iy          k
  1          0.02      6.77e-10       0
  2          0.02      6.77e-10      642
  20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir      V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y    0.044482
Local
* El      V1     (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
     ... Next load cases and/or Combine directives
Stop
```

A.4.6 For Pile Width = 38.1 mm

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.        L          Iy          k
  1         0.02       8.13e-10     0
  2         0.02       8.13e-10     672
  20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
     ... Next load cases and/or Combine directives
Stop
```

A.4.7 For Pile Width = 44.45 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 *kN*.

```
Elements=20
* El.        L          Iy          k
  1        0.02       9.48e-10      0
  2        0.02       9.48e-10     698
 20
E=200e6
Supports=0
* N  code     {code=Y,FIX,SY}

Springs=0
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir      V     {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1       Y     0.044482
Local
* El  V1  (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

A.4.8 For Pile Width = 50.8 *mm*

Title: Analysis of a Short Pile Embedded in Sandy Soil and is Subjected to Lateral Force = 0.044482 kN.

```
Elements=20
* El.          L          Iy         k
  1          0.02       10e-10       0
  2          0.02       10e-10      727
 20
E=200e6
Supports=0
* N  code      {code=Y,FIX,SY}

Springs=0
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}

Load Case: #1
Global
* N      dir       V    {or Nj to Nk  (dN)  dir  V} {dir=Y,M}
  1      Y    0 .044482
Local
* El     V1      (V2)  {or Ej to Ek   V1  (V2)}

Solve
* ... Next load cases
* Combine n1 n2..nk
    ... Next load cases and/or Combine directives
Stop
```

Appendix B

ANALYSIS OF PILES EMBEDDED IN HOMOGENOUS SOIL SUBJECTED TO LATERAL FORCE

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.1 Width of Pile = 6.35 mm and the Length of the model = 350 mm - n_h (initial) = 490 kN/m^3

Table B.1.1 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h(kN/m^3)$
2	0.008889	0.003868	-0.00098	0.003	-0.000868	0.00001177	74
4	0.017793	0.007736	-0.00196	0.005	-0.002736	0.00002354	116
5	0.022241	0.00967	-0.00245	0.006	-0.00367	0.00002943	125
8	0.035585	0.015472	-0.00392	0.009	-0.006472	0.00004708	137
13	0.057826	0.025142	-0.00637	0.015	-0.01014	0.00007651	133
15	0.0.06672	0.02901	-0.00735	0.018	-0.01101	0.00008828	125
17	0.075619	0.03288	-0.00833	0.021	-0.01188	0.0001	119
18	0.080067	0.03481	-0.00882	0.029	-0.00581	0.000106	55
19	0.084516	0.03675	-0.00931	0.036	-0.00075	0.0001118	7
20	0.08896	0.03868	-0.0098	0.045	0.00814	0.0001177	-69
20.8	0.09252	0.040227	-0.0102	0.052	0.01177	0.0001224	-96
21	0.09341	0.04061	-0.0103	0.053	0.01239	0.0001236	-100.

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\%\rvert$
564	88766	0.003	0
606	95456	0.0045	0.05
615	96790	0.0056	0.04
627	98801	0.0088	0.02
623	98023	0.0156	0.06
615	96793	0.0182	0.02
609	95861	0.0208	0.02
545	85784	0.0284	0.06
497	78209	0.0352	0.08
421	66261	0.0456	0.06
394	62009	0.053	0.1
389	61366	0.0539	0.09

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 6.35 mm

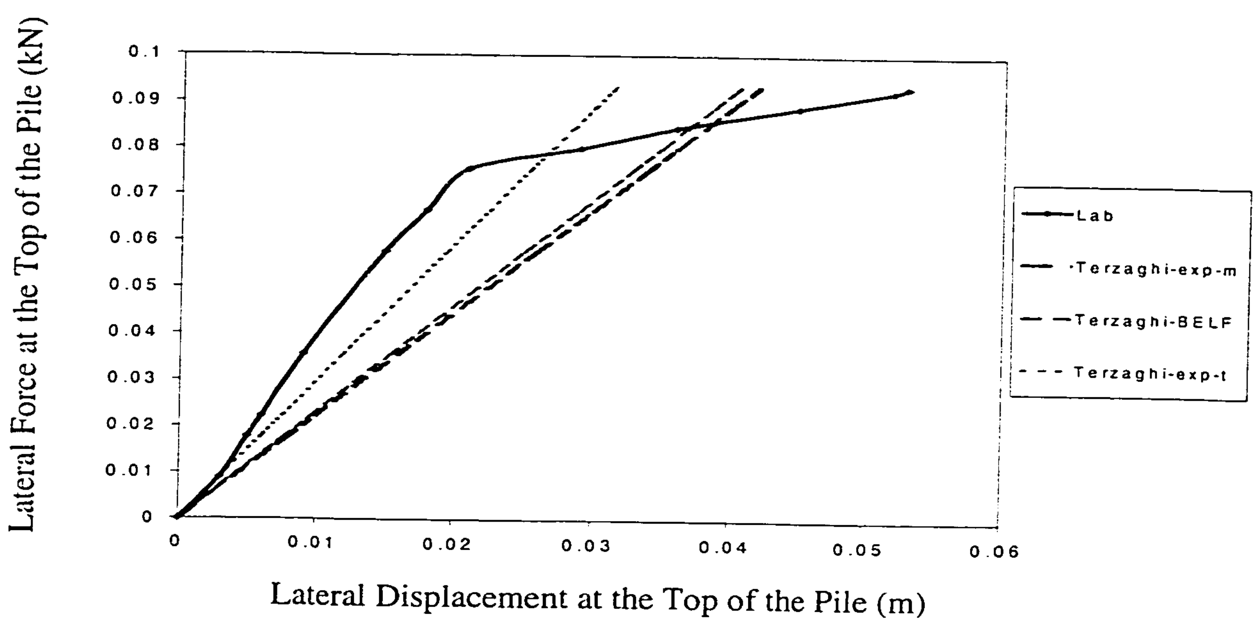


Figure B.1 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

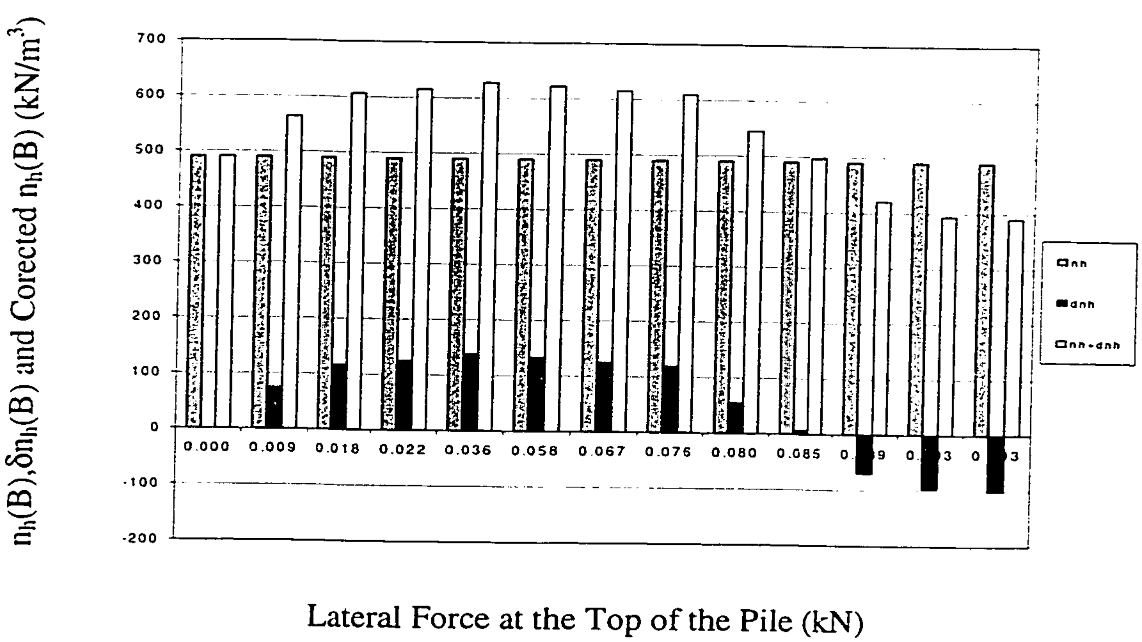


Figure B.2 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.2 Width of Pile = 12.7 mm and the Length of the model = 350 mm - n_h (initial) = 529 kN/m^3

Table B.1.2 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
2	0.008896	0.00338	-0.000956	0.002	-0.00138	0.0000088	157
4	0.0178	0.00676	-0.001912	0.004	-0.00276	0.0000176	157
5	0.02224	0.00845	-0.00239	0.005	-0.00345	0.000022	157
8	0.035585	0.01352	-0.003824	0.008	-0.00552	0.0000352	157
13	0.057783	0.02197	-0.006214	0.012	-0.01	0.0000572	175
15	0.06672	0.02535	-0.00717	0.015	-0.01035	0.000066	157
18	0.080067	0.03042	-0.008604	0.019	-0.01142	0.0000792	144
20	0.08896	0.0338	-0.00956	0.027	-0.0068	0.000088	77
21	0.093412	0.03549	-0.01004	0.037	0.00151	0.0000924	-16
22	0.09786	0.03718	-0.01052	0.044	0.00682	0.0000968	-70
22.8	0.101	0.03853	-0.0109	0.051	0.01247	0.0001	-125
23	0.10231	0.03887	-0.011	0.052	0.01313	0.00001012	-130

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
686	53979	0.00195	0.005
686	53979	0.0036	0.04
686	53979	0.0048	0.02
686	53979	0.0081	0.01
704	55395	0.0125	0.05
686	53979	0.0156	0.06
673	52983	0.0188	0.02
606	47713	0.0279	0.09
513	40342	0.038	0.1
459	36081	0.0449	0.09
404	31810	0.0521	0.11
399	31413	0.0532	0.12

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 12.7 mm

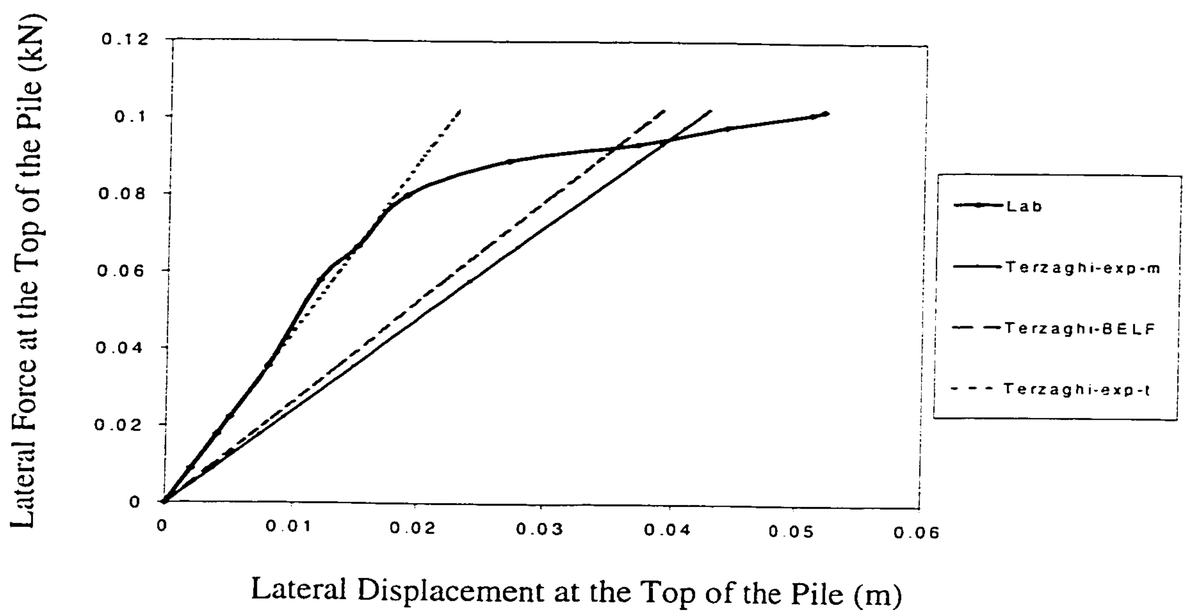


Figure B.3 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

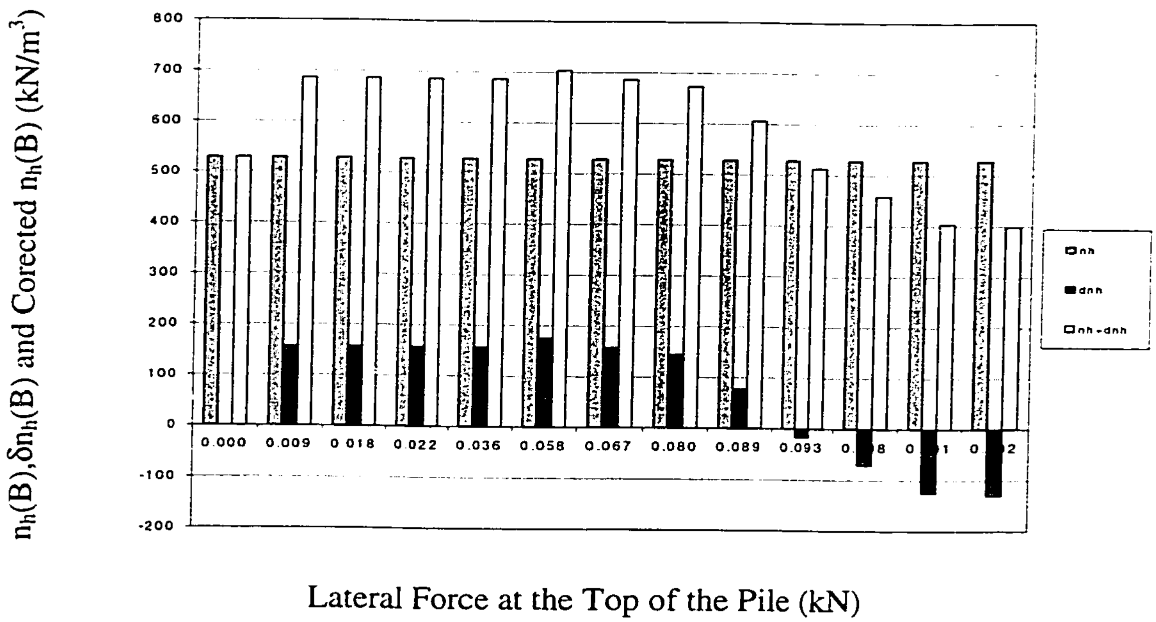


Figure B.4 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.3 Width of Pile = 19.05 mm and the Length of the model = 350 mm - n_h (initial) = 736 kN/m^3

Table B.1.3 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
6	0.02668	0.00726	-0.00207	0.004	-0.00326	0.00001358	240
10	0.04448	0.0121	-0.00345	0.007	-0.0051	0.00002264	225
17	0.07562	0.02057	-0.005865	0.014	-0.00657	0.0000385	171
20	0.08896	0.0242	-0.0069	0.023	-0.0012	0.00004528	3
22	0.09786	0.02662	-0.00759	0.029	0.00238	0.00007981	-30
23	0.10231	0.02783	-0.007935	0.031	0.00317	0.00005207	-61
24	0.10676	0.02904	-0.00828	0.038	0.0096	0.00005433	-176
25	0.11121	0.03025	-0.008625	0.04	0.00975	0.0000566	-172
26	0.11565	0.03146	-0.00897	0.046	0.01454	0.00005886	-247
26.8	0.11922	0.03243	-0.009246	0.05	0.01757	0.00006067	-290
27	0.1201	0.03267	-0.009315	0.051	0.01833	0.00006113	-300

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
976	51224	0.0037	0.03
961	50448	0.0066	0.04
907	47580	0.0135	0.05
739	38761	0.0226	0.04
706	37057	0.0291	0.01
675	35427	0.0316	0.06
560	29347	0.0388	0.08
564	29580	0.0411	0.11
489	25655	0.0472	0.12
446	23420	0.0515	0.15
436	22882	0.0523	0.13

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 19.5 mm

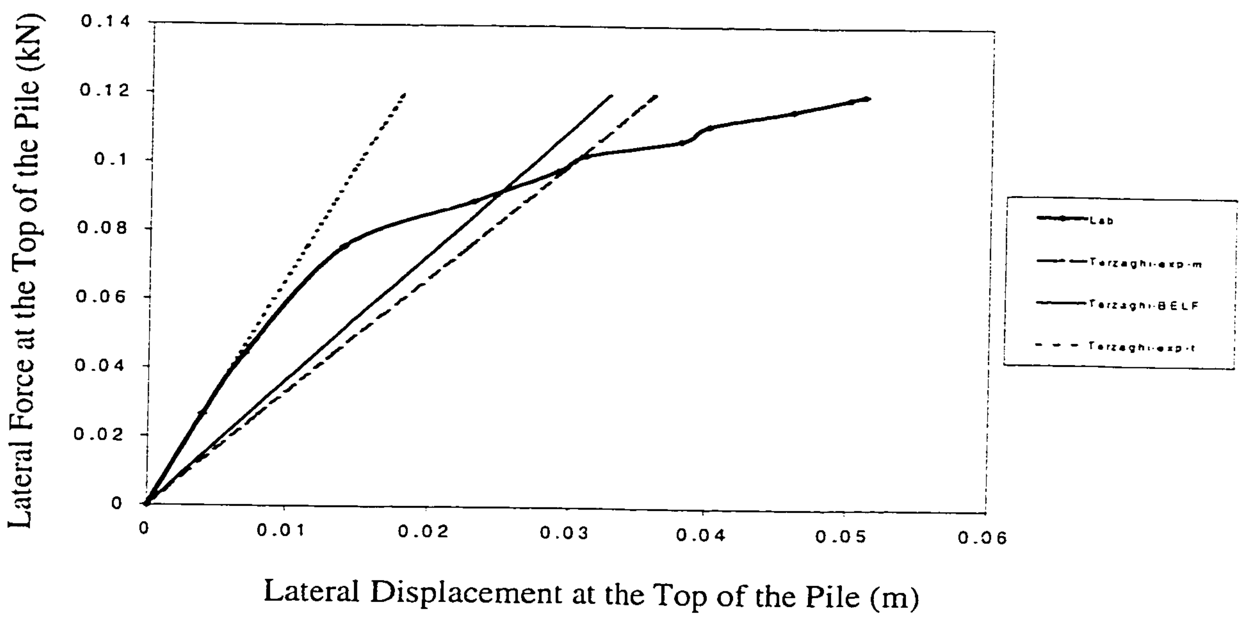


Figure B.5 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

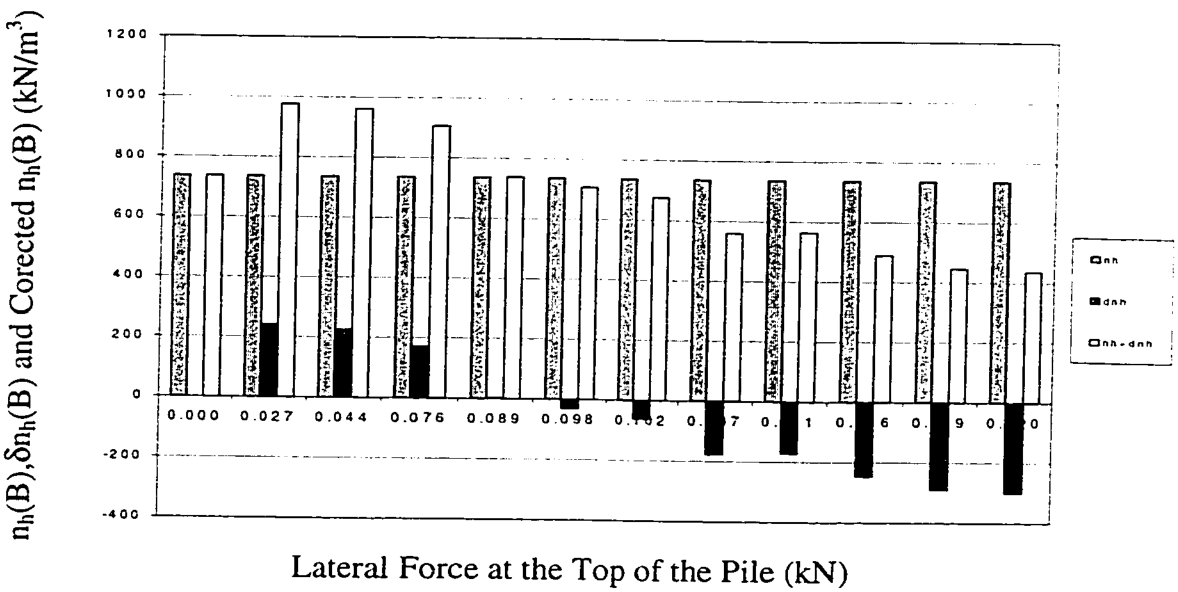


Figure B.6 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.4 Width of Pile = 25.4 mm and the Length of the model = 350 mm - n_h (initial) = 738 kN/m^3

Table B.1.4 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
10	0.04448	0.0118	-0.00342	0.005	-0.0068	0.00002148	317
14	0.06227	0.01652	-0.00479	0.008	-0.00852	0.00003007	283
17	0.07562	0.02006	-0.00581	0.012	-0.00806	0.00003652	221
20	0.08896	0.0236	-0.00684	0.019	-0.0046	0.00004296	107
22	0.09786	0.0236	-0.00752	0.026	0.0024	0.00004726	-51
23	0.10231	0.02714	-0.00787	0.028	0.00086	0.00004940	-17
25	0.11121	0.0295	-0.00855	0.035	0.0055	0.00005370	-102
26	0.11565	0.03068	-0.00889	0.039	0.00832	0.00005585	-149
27	0.1201	0.03186	-0.00923	0.044	0.01214	0.00005800	-209
28	0.12455	0.03304	-0.00958	0.048	0.01496	0.00006014	-249
28.8	0.12811	0.033984	-0.00985	0.049	0.015016	0.00006186	-243
29	0.129	0.03422	-0.00992	0.05	0.01578	0.00006229	-253

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
1055	41517	0.0045	0.05
1021	40208	0.0075	0.05
959	37744	0.0116	0.04
845	33269	0.0187	0.03
687	27054	0.0266	0.06
721	28368	0.0282	0.02
636	25021	0.0355	0.05
589	23188	0.0408	0.18
529	20812	0.0451	0.11
489	19261	0.0496	0.16
495	19497	0.05	0.1
485	19080	0.052	0.2

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 25.4 mm

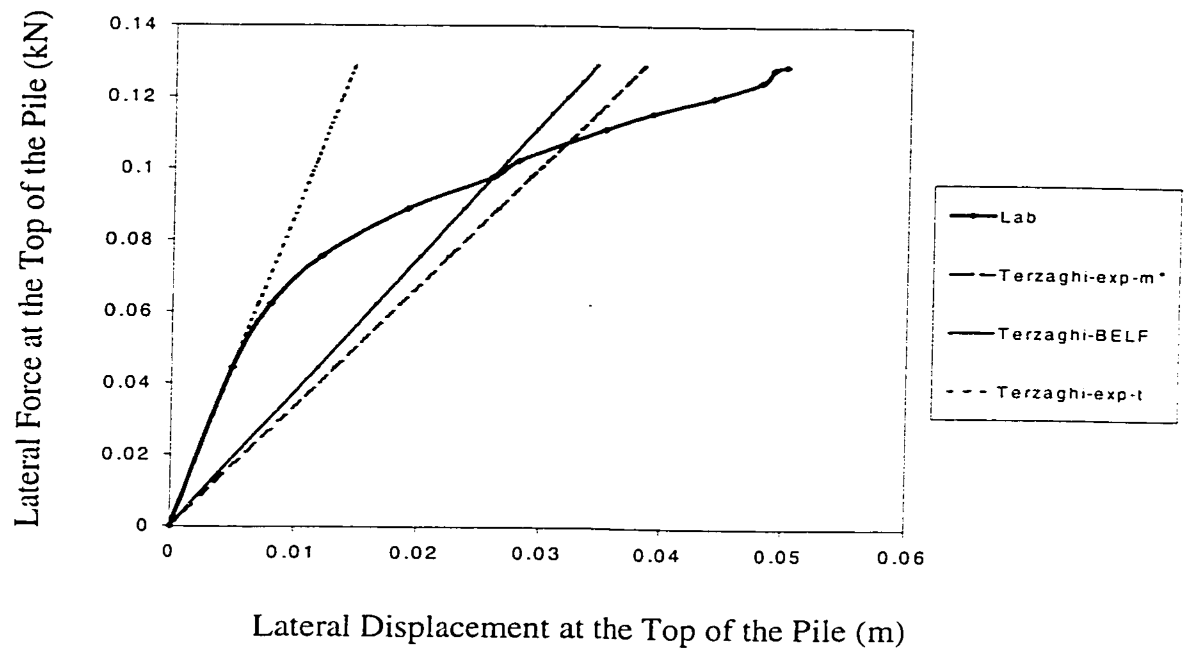


Figure B.7 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

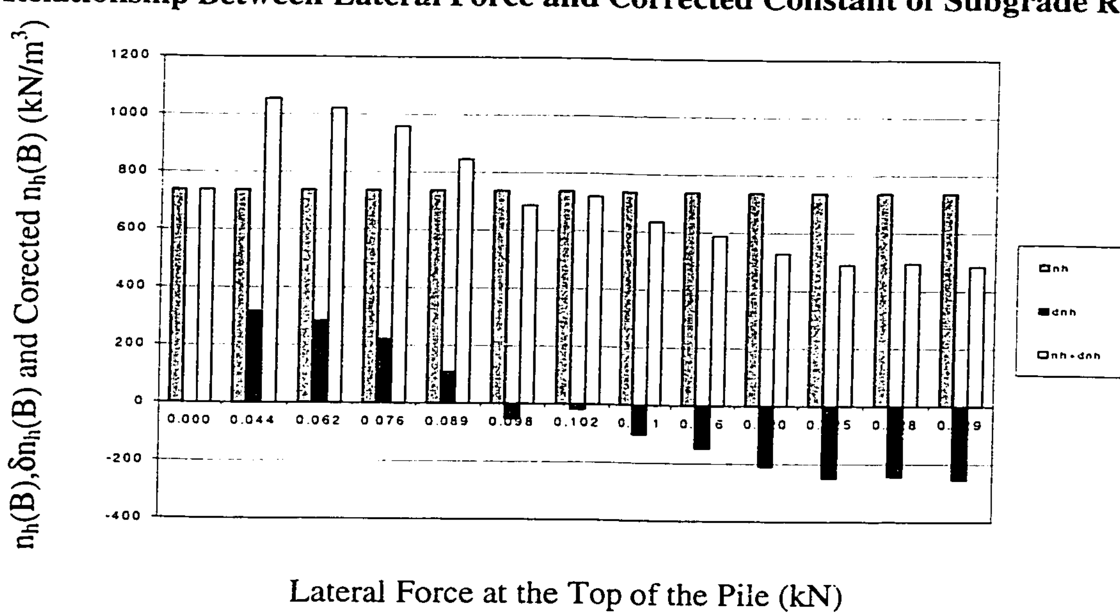


Figure B.8 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.5 Width of Pile = 31.75 mm and the Length of the model = 350 mm - n_h (initial) = 1225 kN/m^3

Table B.1.5 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
10	0.04448	0.007	-0.0021	0.004	-0.003	0.000007534	398
14	0.06227	0.0098	-0.00294	0.006	-0.0038	0.00001055	360
18	0.0801	0.0126	-0.00378	0.011	-0.0016	0.00001356	118
20	0.08896	0.014	-0.0042	0.015	0.001	0.00001507	-66
22	0.0978	0.0154	-0.00462	0.02	0.0046	0.00001657	-277
25	0.11121	0.0175	-0.00525	0.025	0.0075	0.00001884	-398
26	0.11565	0.0182	-0.00546	0.029	0.0108	0.00001959	-551
27	0.1201	0.0189	-0.00567	0.031	0.0121	0.00002034	-595
28	0.12455	0.0196	-0.00588	0.032	0.0124	0.00002110	-588
29	0.129	0.0203	-0.00609	0.035	0.0147	0.00002185	-673
29.5	0.13122	0.02065	-0.0062	0.037	0.01635	0.00002223	-736
29.8	0.13255	0.02086	-0.00626	0.039	0.01814	0.00002245	-808
30	0.13344	0.021	-0.0063	0.04	0.019	0.000022602	-841

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
1623	51118	0.0039	0.01
1585	49923	0.0062	0.02
1343	42292	0.0116	0.06
1159	36486	0.0148	0.02
947	29835	0.0227	0.27
827	26035	0.0256	0.06
673	21211	0.0287	0.03
630	19841	0.031	0.0
637	20062	0.0323	0.03
552	17385	0.0359	0.09
489	15406	0.0382	0.12
417	13128	0.0405	0.15
384	12099	0.0417	0.17

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 31.75 mm

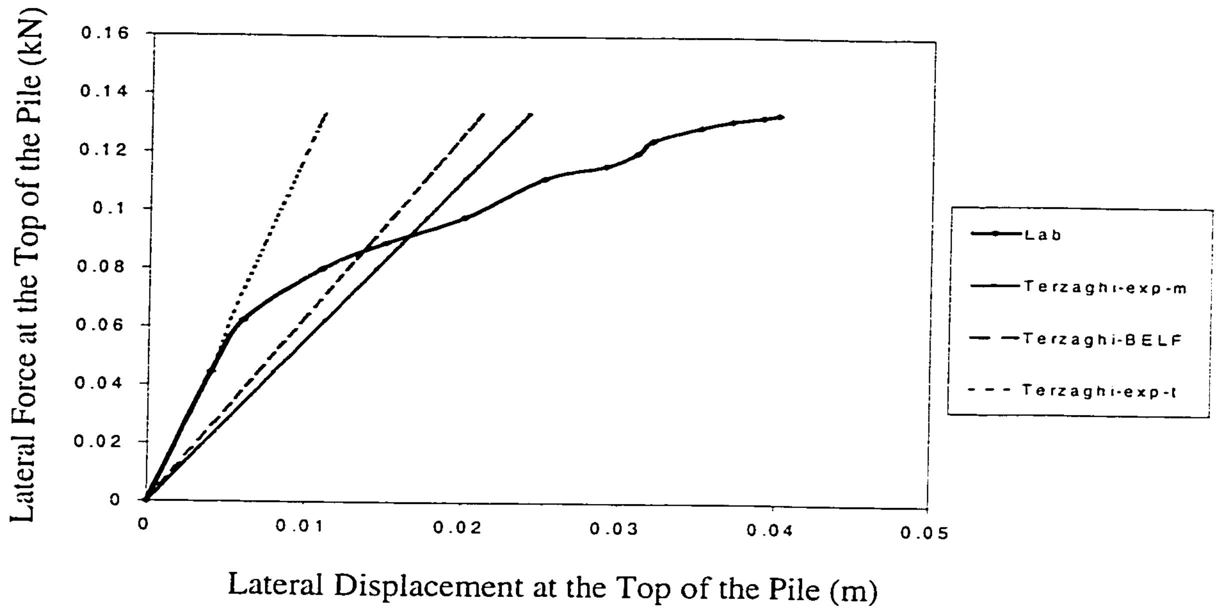


Figure B.9 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

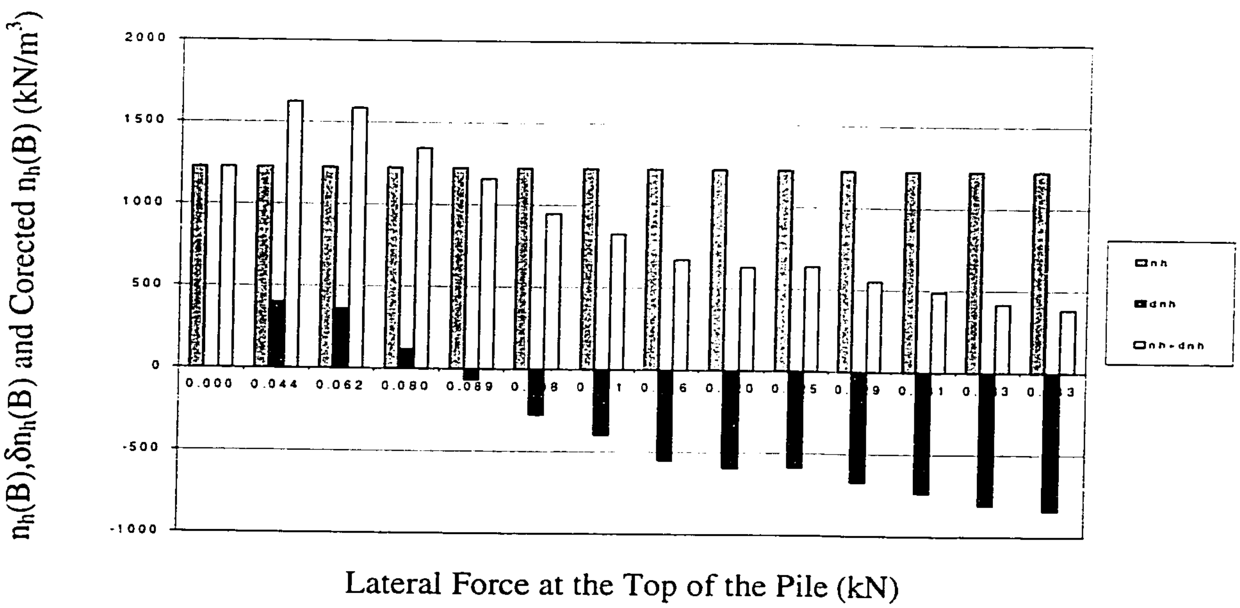


Figure B.10 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.6 Width of Pile = 38.1 mm and the Length of the model = 350 mm - n_h (initial) = 1762 kN/m^3

Table B.1.6 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
10	0.04448	0.005	-0.00142	0.003	-0.002	0.000007534	265
14	0.06227	0.007	-0.00199	0.005	-0.002	0.00001055	190
18	0.08006	0.009	-0.00256	0.009	0	0.00001356	0
21	0.09341	0.0105	-0.00298	0.013	0.0025	0.00001582	-158
22	0.09786	0.011	-0.00312	0.014	0.003	0.00001657	-181
25	0.11121	0.0125	-0.00355	0.019	0.0065	0.00001884	-345
26	0.11565	0.013	-0.00369	0.02	0.007	0.00001959	-35
28	0.12455	0.014	-0.00398	0.024	0.01	0.00002110	-474
29	0.129	0.0145	-0.00412	0.025	0.0105	0.00002185	-481
30	0.1334	0.015	-0.00426	0.027	0.012	0.00002260	-531
30.8	0.137	0.0154	-0.00437	0.029	0.0136	0.00002320	-586
31	0.1379	0.0155	-0.0044	0.03	0.0145	0.00002336	-621

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
2028	53222	0.0031	0.01
1952	51231	0.0048	0.02
1762	46254	0.009	0.0
1604	42107	0.0138	0.08
1581	41504	0.0145	0.05
1417	37196	0.020	0.1
1405	36875	0.0205	0.05
1288	33812	0.0249	0.09
1282	33641	0.0261	0.11
1231	32319	0.0286	0.16
1176	30871	0.030	0.1
1141	29959	0.0315	0.15

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 38.1 mm

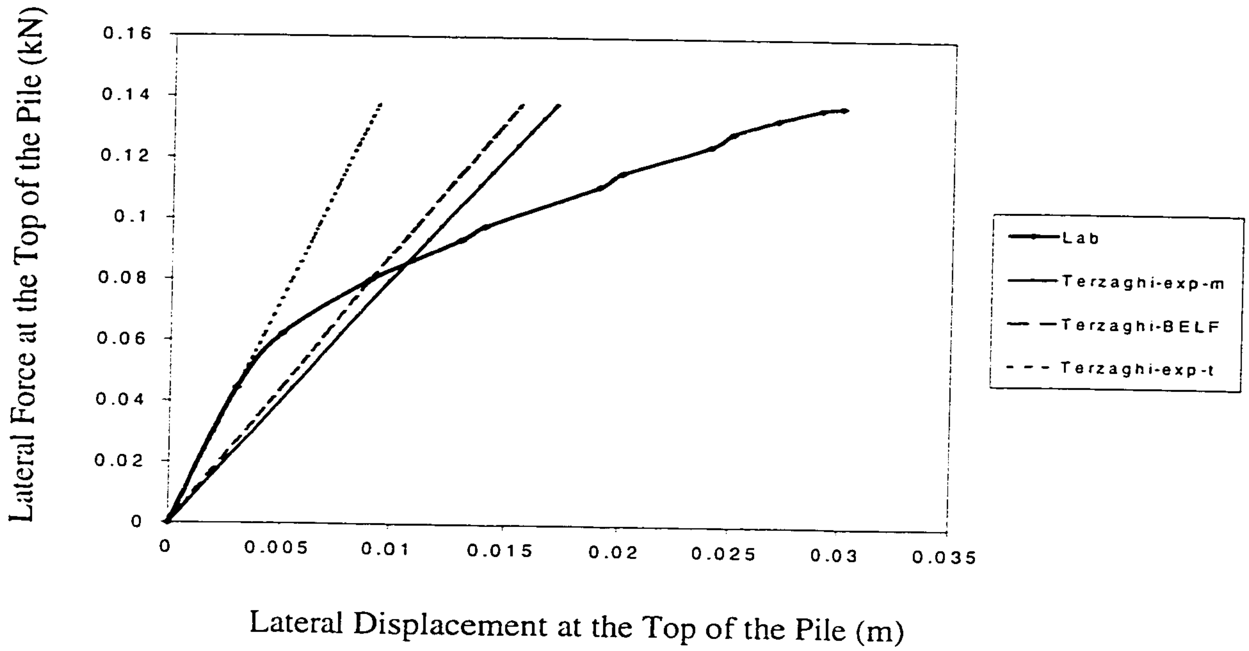


Figure B.11 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

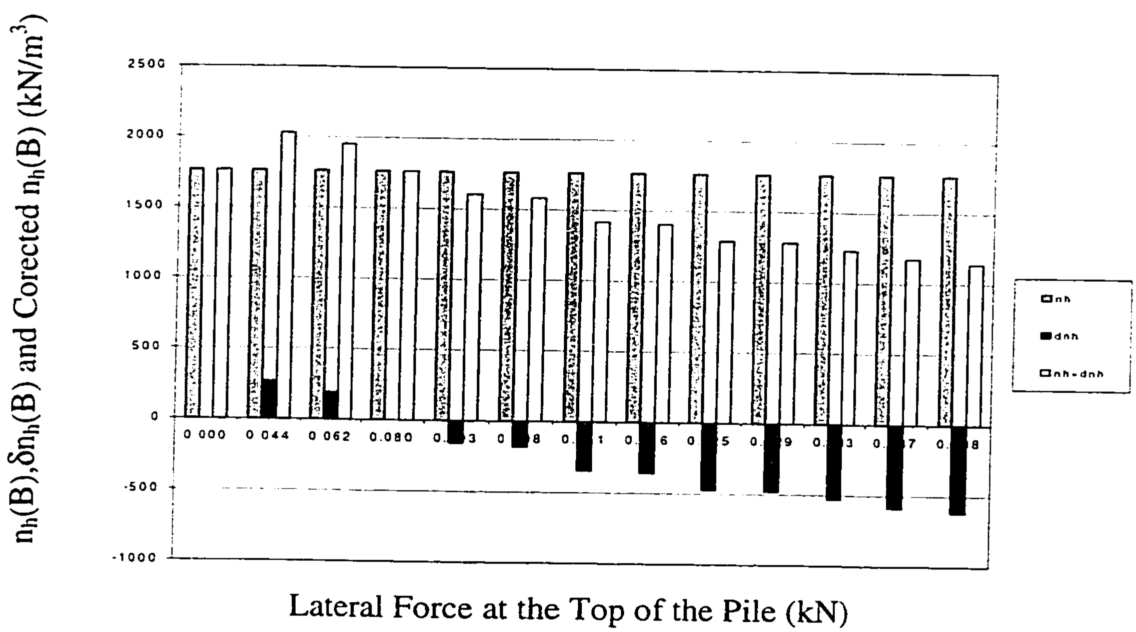


Figure B.12 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.7 Width of Pile = 44.45 mm and the Length of the model = 350 mm - n_h (initial) = 2798 kN/m^3

Table B.1.7 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h (kN/m^3)$
11	0.04893	0.005035	-0.00141	0.003	-0.00203	3.57104E-06	570
14	0.06227	0.006408	-0.0018	0.004	-0.00241	4.54496E-06	530
19	0.08451	0.008696	-0.00244	0.008	-0.0007	6.16816E-06	113
24	0.10675	0.010985	-0.00309	0.012	0.001015	7.79136E-06	-130
25	0.11121	0.011443	-0.00322	0.013	0.001558	0.000008116	-192
27	0.1201	0.012358	-0.00347	0.016	0.003642	8.76528E-06	-416
29	0.129	0.013273	-0.00373	0.019	0.005727	9.41456E-06	-608
30	0.13344	0.013731	-0.00386	0.021	0.007269	9.7392E-06	-746
32	0.14234	0.014646	-0.00412	0.024	0.009354	1.03885E-05	-900
32.5	0.14456	0.014875	-0.00418	0.025	0.010125	1.05508E-05	-960
32.8	0.1459	0.015013	-0.00422	0.026	0.010987	1.06482E-05	-1032
33	0.14679	0.015104	-0.00424	0.027	0.011896	1.07131E-05	-1110

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor} (kN/m^3)$	$n_{h\,cor} (kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
3368	75757	0.0027	0.03
3328	74857	0.0038	0.02
2911	65478	0.0081	0.01
2667	60007	0.0125	0.05
2606	58621	0.0139	0.09
2382	53591	0.0167	0.07
2189	49254	0.0189	0.01
2051	46148	0.0211	0.01
1897	42683	0.0249	0.09
1838	41350	0.0261	0.11
1766	39725	0.0275	0.15
1687	37958	0.0288	0.18

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 44.45 mm

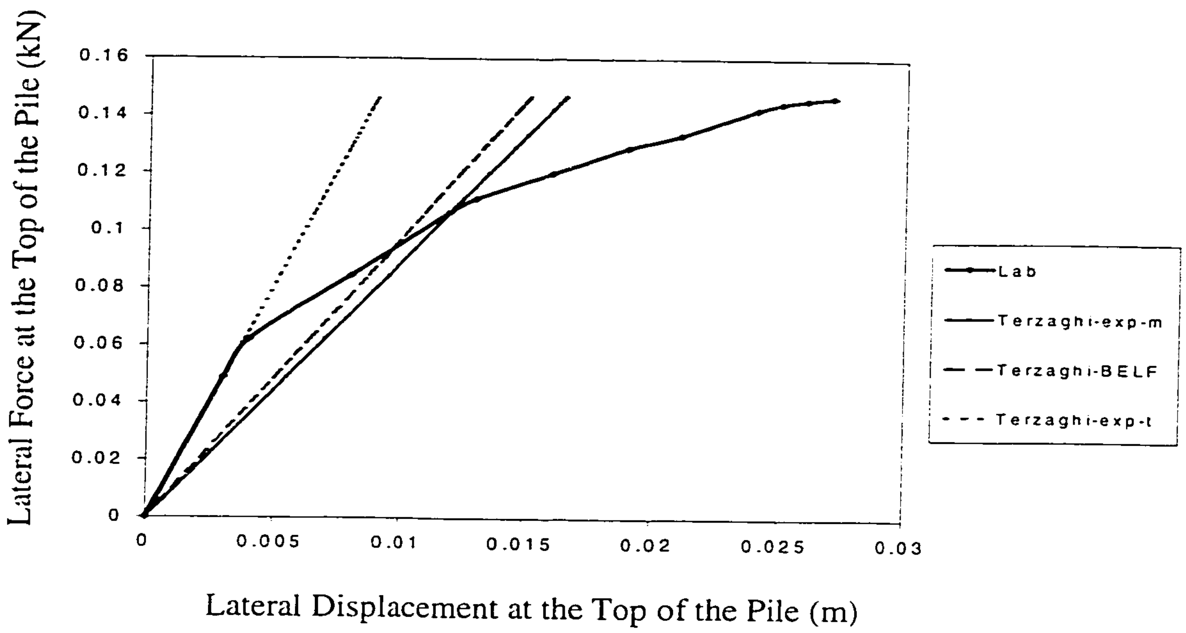


Figure B.13 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

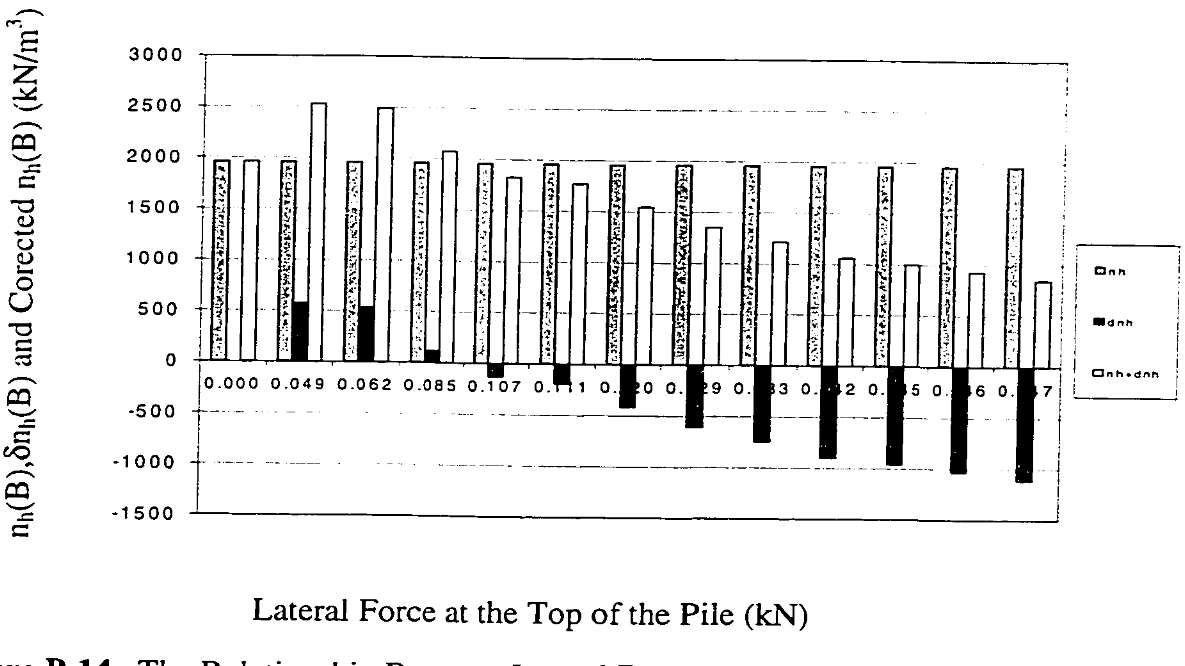


Figure B.14 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.1 Analysis of Piles Embedded in Sandy Soil Subjected to Lateral Force

B.1 Terzaghi's Method Used for Determination of Initial Constant of Horizontal Subgrade Reaction

B.1.8 Width of Pile = 50.8 mm and the Length of the model = 350 mm - n_h (initial) = 3260 kN/m^3

Table B.1.8 Calculation of Variation of Constant of Horizontal Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta n_h(kN/m^3)$
11	0.04893	0.004587	-0.00128	0.002	-0.00259	2.96494E-06	873
14	0.06227	0.005838	-0.00163	0.003	-0.00284	3.77356E-06	752
19	0.08451	0.007923	-0.00222	0.006	-0.00192	5.12126E-06	375
23	0.1023	0.009591	-0.00268	0.01	0.000409	6.1995E-06	-66
25	0.11121	0.010425	-0.00292	0.012	0.001575	6.7686E-06	-233
29	0.129	0.01209	-0.00338	0.015	0.00291	7.8168E-06	-372
30	0.13344	0.01251	-0.0035	0.016	0.00349	8.0864E-06	-432
31	0.13789	0.012927	-0.00361	0.018	0.005073	8.3559E-06	-607
33	0.14679	0.01376	-0.00385	0.021	0.00724	0.000008895	-814
34	0.15124	0.014178	-0.00396	0.026	0.011822	9.1646E-06	-1290
34.8	0.15479	0.014512	-0.00402	0.028	0.013488	9.3802E-06	-1438
35	0.15568	0.014595	-0.00408	0.029	0.014405	9.4341E-06	-1527

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$n_{h\,cor}(kN/m^3)$	$n_{h\,cor}(kN/m^4)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
4133	81354	0.0021	0.01
4012	78982	0.0033	0.03
3636	71569	0.0068	0.08
3194	62879	0.0111	0.11
3028	59597	0.0126	0.06
2888	56849	0.0154	0.04
2829	55682	0.0168	0.08
2653	52227	0.0179	0.01
2446	48155	0.022	0.1
1970	38785	0.0271	0.11
1822	35871	0.0285	0.05
1733	34121	0.03	0.1

The Relationship Between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 50.8 mm

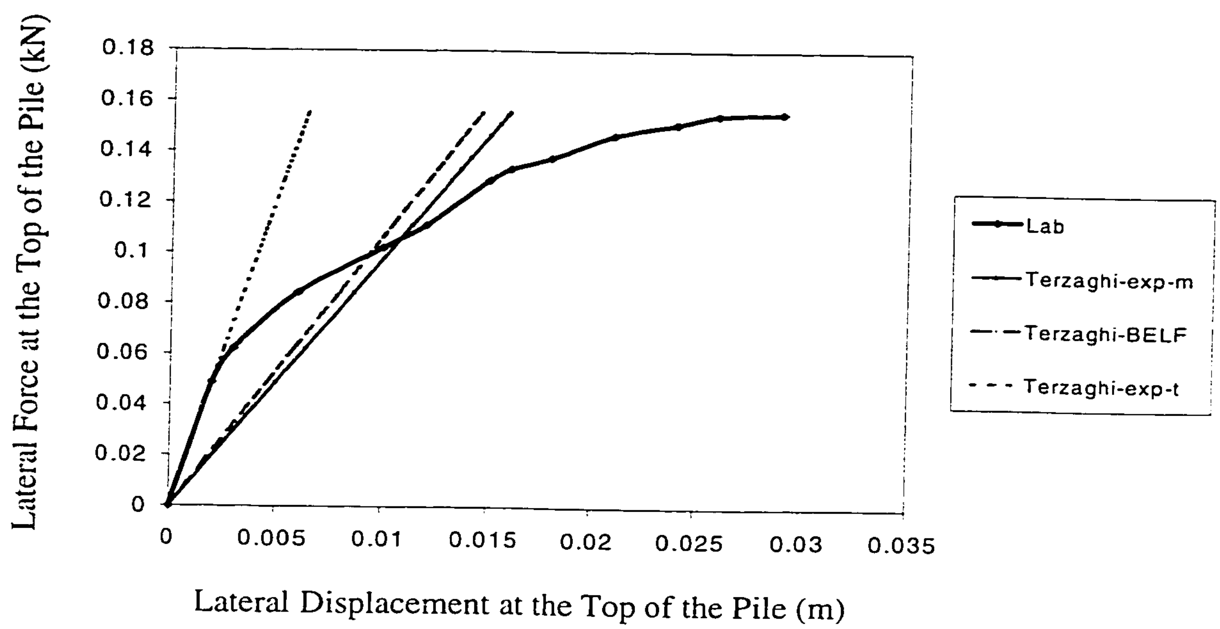


Figure B.15 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Constant of Subgrade Reaction.

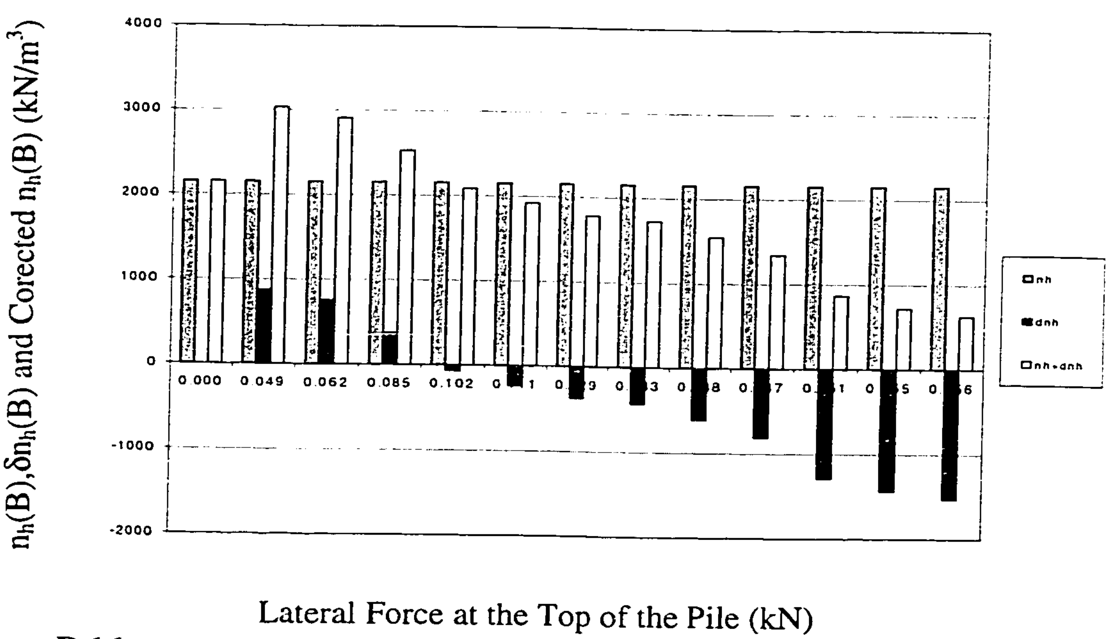


Figure B.16 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.1 Width of Pile = 6.35 mm and the length of the model = 400 mm - $k_h(B)$ (initial)= 83 kN/m^2

Table B.2.1.1 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.01191	-0.0039	0.01	-0.00191	0.000221	9
20	0.08896	0.01588	-0.0052	0.013	-0.00288	0.0002946	10
25	0.11120	0.01985	-0.0065	0.020	0.00015	0.0003683	-0.5
26	0.115653	0.02064	-0.00676	0.027	0.006356	0.000383	-17
27	0.1201	0.02144	-0.00702	0.034	0.012562	0.0003978	-14
28	0.12455	0.02223	-0.00728	0.037	0.014768	0.0004125	-36
28.7	0.12766	0.02278	-0.00746	0.038	0.015212	0.0004228	-36
29	0.129	0.02303	-0.00754	0.041	0.017974	0.0004273	-42
29.2	0.13	0.02318	-0.007592	0.042	0.018815	0.00043	-44

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
92	14355	0.01	0
93	14534	0.0132	0.02
82.5	12930	0.0199	0.01
66	10380	0.0268	0.02
69	10856	0.0335	0.05
47	7357	0.0368	0.02
47	7328	0.0377	0.03
41	6370	0.0407	0.03
39	6104	0.0414	0.06

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 6.35 mm

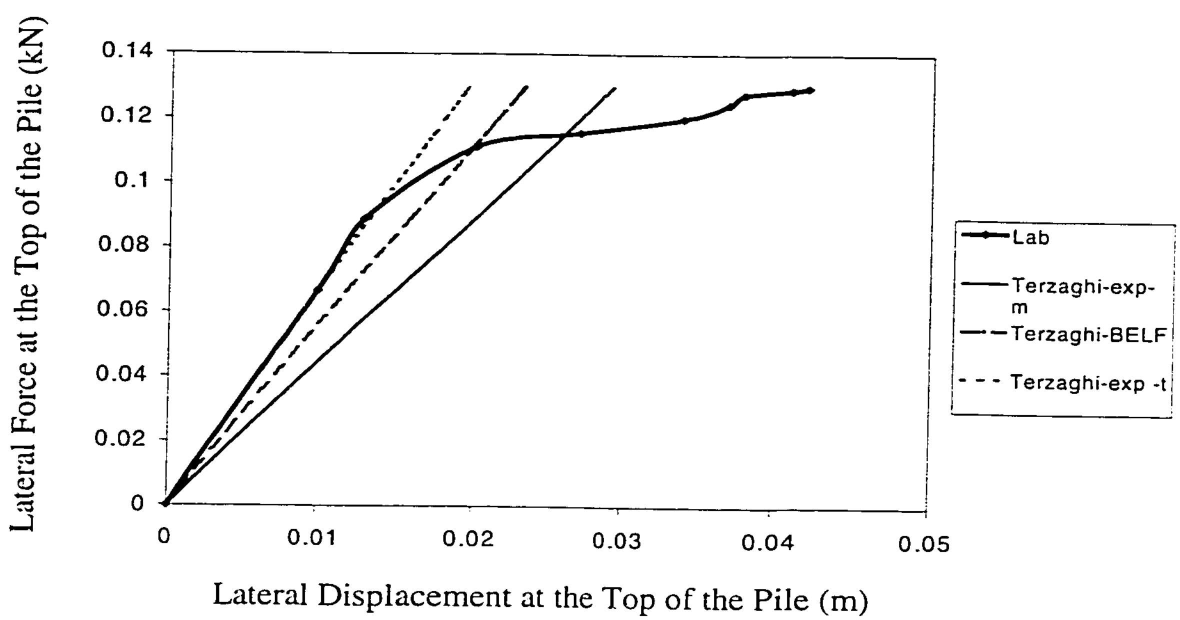


Figure B.17 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

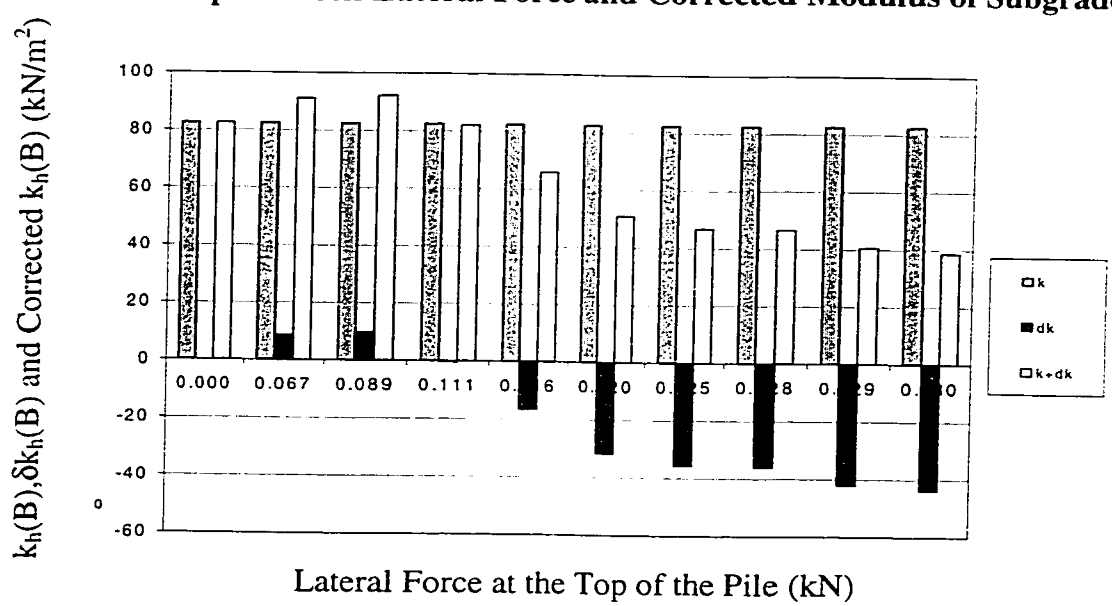


Figure B.18 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.2 Width of Pile = 12.7 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 103 kN/m^2

Table B.2.1.2. Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.009	-0.00339	0.007	-0.002	0.0001239	16
20	0.08896	0.012	-0.00452	0.011	-0.001	0.0001652	6
25	0.11121	0.015	-0.00565	0.015	0	0.0002065	0.0
27	0.1201	0.0162	-0.006102	0.019	0.0028	0.0002230	-13
28	0.12455	0.0168	-0.006328	0.021	0.0042	0.0002313	-18
29	0.129	0.0174	-0.006554	0.023	0.0056	0.0002395	-23
30	0.1334	0.018	-0.00678	0.025	0.007	0.0002478	-28
33	0.14679	0.0198	-0.007458	0.029	0.0092	0.0002726	-34
34	0.15124	0.0204	-0.007684	0.03	0.0096	0.0002808	-34
35	0.15568	0.021	-0.00791	0.031	0.01	0.0002891	-35
36	0.16013	0.0216	-0.008136	0.036	0.0144	0.0002974	-48
36.5	0.162359	0.0219	-0.008249	0.039	0.0171	0.0003015	-57
36.8	0.16369	0.02208	-0.008317	0.040	0.0192	0.000304	-59
37	0.16458	0.0222	-0.008362	0.041	0.0188	0.0003055	-62

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
119	9392	0.0071	0.01
109	8597	0.011	0.0
103	8121	0.015	0.0
90	7132	0.0192	0.02
85	6691	0.0211	0.01
80	6280	0.0234	0.04
75	5897	0.0254	0.04
69	5464	0.0292	0.02
69	5429	0.0307	0.07
68	5397	0.0311	0.01
55	4306	0.0367	0.07
46	3655	0.0397	0.07
44	3480	0.041	0.1
41	3276	0.0428	0.18

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 12.7 mm

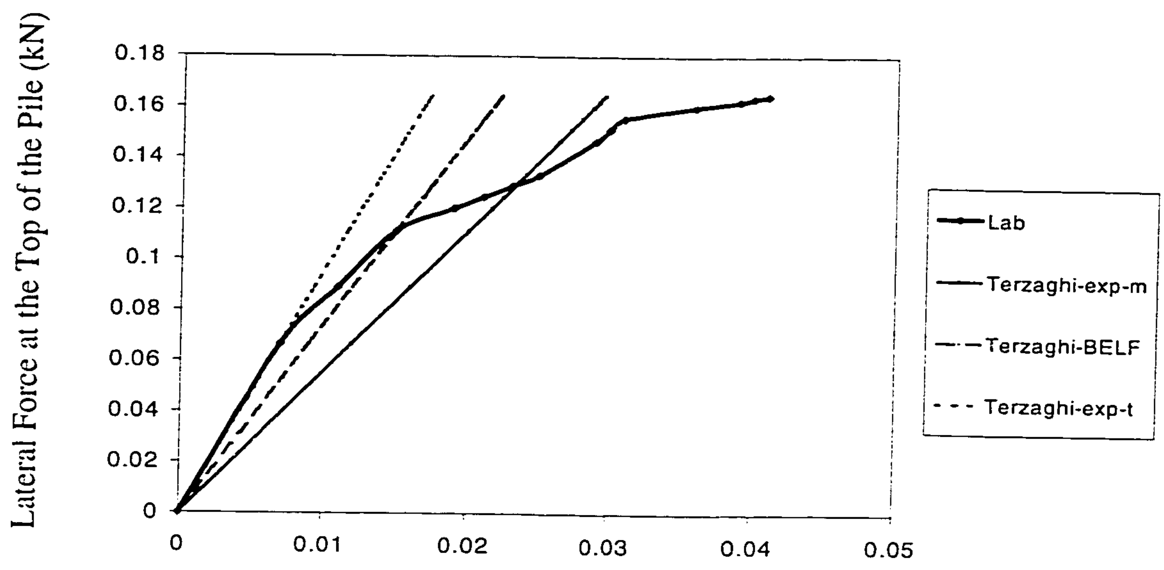


Figure B.19 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

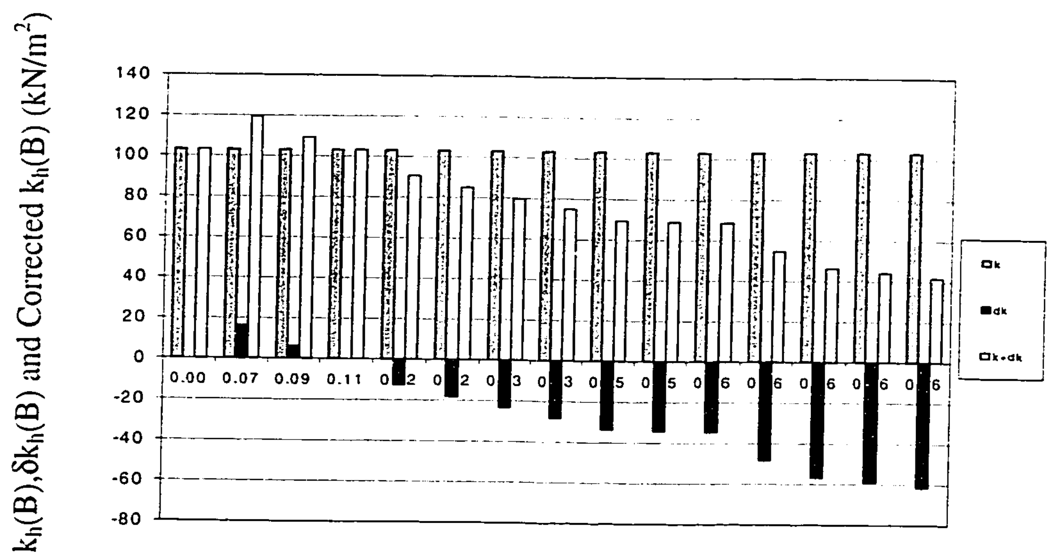


Figure B.20 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.3 Width of Pile = 19.05 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 146 kN/m^2

Table B.2.1.3 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.0063	-0.00243	0.0055	-0.0008	0.0000605	13
17	0.07562	0.00714	-0.00275	0.006	-0.00114	0.0000686	17
22	0.09786	0.00924	-0.003564	0.01	0.00076	0.0000887	-9
27	0.1201	0.01134	-0.004374	0.014	0.00266	0.0001089	-24
30	0.1334	0.0126	-0.00486	0.017	0.0044	0.000121	-36
33	0.1468	0.01386	-0.005346	0.021	0.00714	0.0001331	-54
36	0.16013	0.01512	-0.005832	0.025	0.00988	0.0001452	-68
37	0.16458	0.01554	-0.005994	0.026	0.01046	0.000149	-70
39	0.17348	0.01638	-0.006318	0.029	0.01262	0.0001573	-80
39.5	0.1757	0.01659	-0.0064	0.031	0.01441	0.000159	-91
40	0.17793	0.0168	-0.00648	0.033	0.0162	0.0001613	-100
40.8	0.1815	0.017136	-0.00661	0.035	0.01786	0.0001645	-109
41	0.18237	0.01722	-0.00664	0.036	0.01878	0.0001654	-114

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
159	8338	0.00572	0.022
163	8517	0.0062	0.02
137	7194	0.0107	0.07
121	6362	0.0139	0.01
110	5735	0.0168	0.02
92	4828	0.0214	0.04
78	4072	0.0241	0.09
76	3959	0.0255	0.05
66	3433	0.0296	0.06
55	2887	0.0307	0.03
46	2372	0.034	0.1
37	1945	0.0359	0.09
32	1684	0.037	0.1

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 19.05 mm

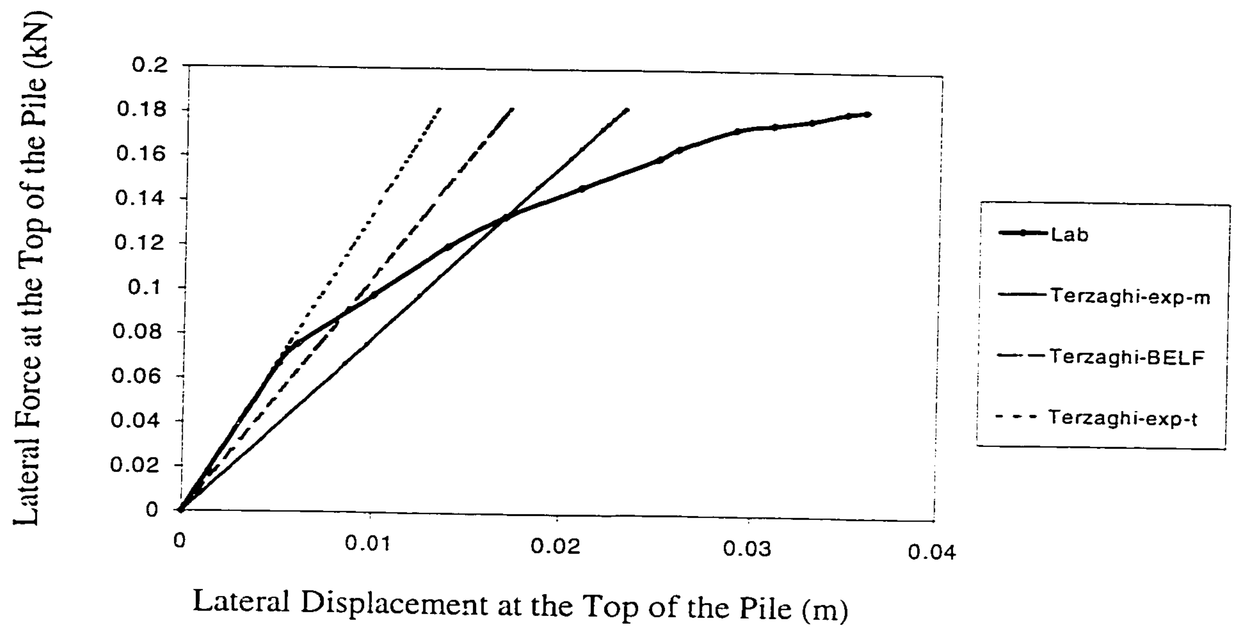


Figure B.21 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

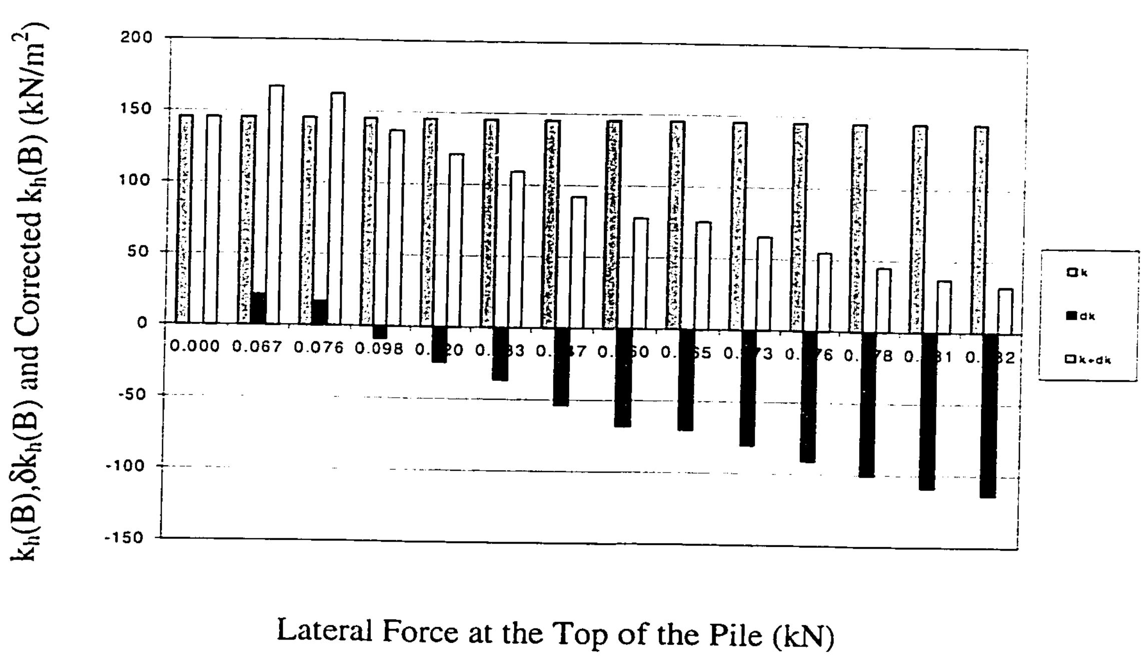


Figure B.22 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.4 Width of Pile = 25.4 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 155 kN/m^2

Table B.2.1.4 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.0667	0.00058	-0.00235	0.004	-0.00181	0.0000513	35
18	0.0801	0.007	-0.00282	0.005	-0.002	0.00006156	32
23	0.10231	0.009	-0.0036	0.009	0.0	0.0000787	0.0
28	0.1245	0.011	-0.00439	0.014	0.003	0.00009576	-31
30	0.13344	0.01163	-0.0047	0.016	0.00437	0.0001026	-43
33	0.1468	0.0128	-0.00517	0.019	0.0062	0.0001129	-55
36	0.16013	0.014	-0.00564	0.022	0.008	0.000123	-65
40	0.1779	0.0155	-0.00627	0.026	0.0105	0.0001368	-77
42	0.1868	0.01628	-0.00658	0.028	0.01172	0.0001436	-82
43	0.19127	0.01667	-0.00674	0.029	0.01233	0.0001471	-84
44	0.19572	0.01705	-0.00689	0.033	0.01595	0.0001505	-106
44.5	0.1977	0.01725	-0.00697	0.036	0.01875	0.0001522	-123
45	0.2002	0.01744	-0.00705	0.037	0.01956	0.000154	-127

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
190	7483	0.0038	0.02
187	7370	0.00482	0.018
155	6091	0.009	0.0
124	4857	0.0146	0.06
112	4414	0.0169	0.09
100	3929	0.0192	0.02
90	3530	0.0224	0.04
78	3069	0.0262	0.02
73	2878	0.0278	0.02
71	2791	0.0291	0.01
49	1918	0.034	0.1
32	1241	0.0369	0.09
28	1090	0.0381	0.11

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 25.4 mm

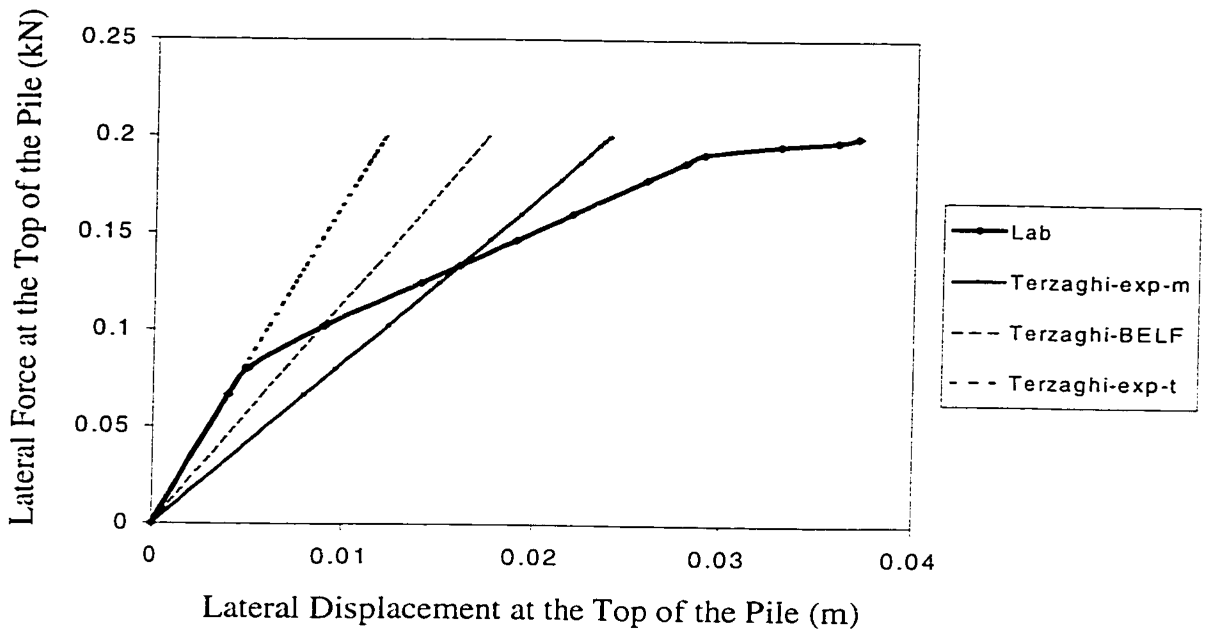


Figure B.23 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

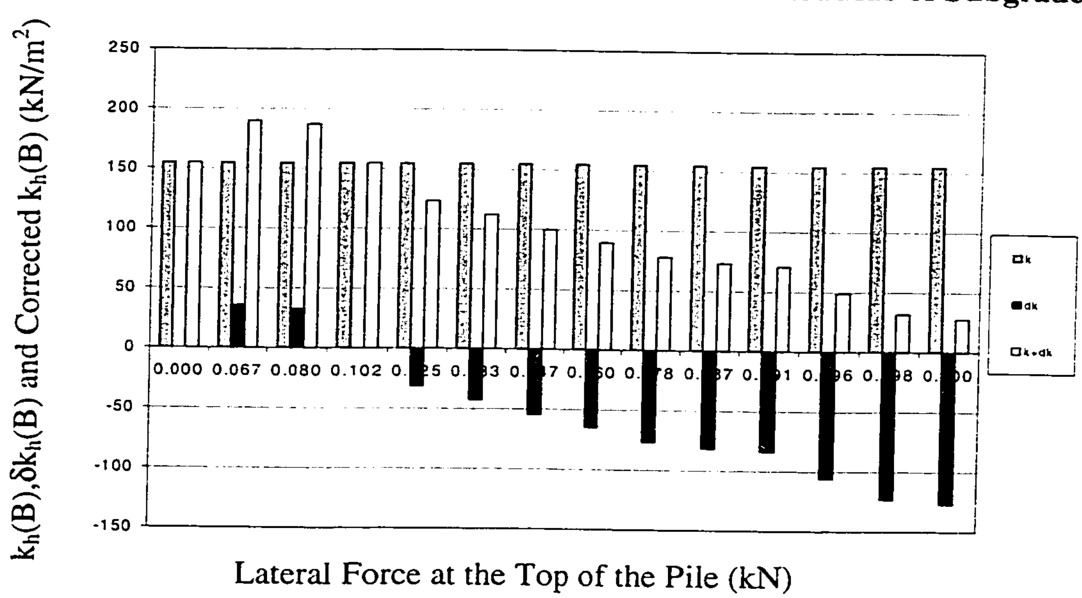


Figure B.24 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.5 Width of Pile = 31.75 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 157 kN/m^2

Table B.2.1.5 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.0801	0.00675	-0.00283	0.004	-0.00275	0.00005742	48
25	0.11121	0.009375	-0.00393	0.009	-0.000375	0.00007975	5
28	0.1245	0.0105	-0.0044	0.011	0.0005	0.00008932	-6
32	0.14234	0.012	-0.005024	0.014	0.002	0.0001021	-20
37	0.16458	0.01387	-0.00581	0.019	0.00513	0.000118	-43
40	0.17793	0.015	-0.00628	0.021	0.006	0.0001276	-42
43	0.19127	0.01613	-0.006751	0.024	0.00787	0.0001372	-57
45	0.2002	0.01687	-0.007065	0.027	0.01013	0.0001436	-71
47	0.2091	0.01763	-0.00738	0.028	0.0104	0.00015	-69
49	0.21796	0.0184	-0.007693	0.030	0.0116	0.000156	-74
50	0.22241	0.01875	-0.00785	0.035	0.01625	0.0001595	-102
51	0.22685	0.01913	-0.00801	0.037	0.01787	0.0001627	-110
51.7	0.22997	0.0194	-0.00812	0.039	0.0196	0.000165	-119
52	0.2313	0.0195	-0.008164	0.040	0.0205	0.000166	-123

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
205	6459	0.004	0
162	5098	0.0093	0.03
151	4774	0.0108	0.02
137	4333	0.0138	0.02
114	3581	0.0192	0.02
115	3627	0.021	0
100	3144	0.0246	0.06
86	2728	0.0279	0.09
88	2767	0.0288	0.08
83	2608	0.031	0.1
55	1741	0.0359	0.09
47	1491	0.0376	0.06
38	1209	0.041	0.2
34	1061	0.0416	0.16

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 31.75 mm

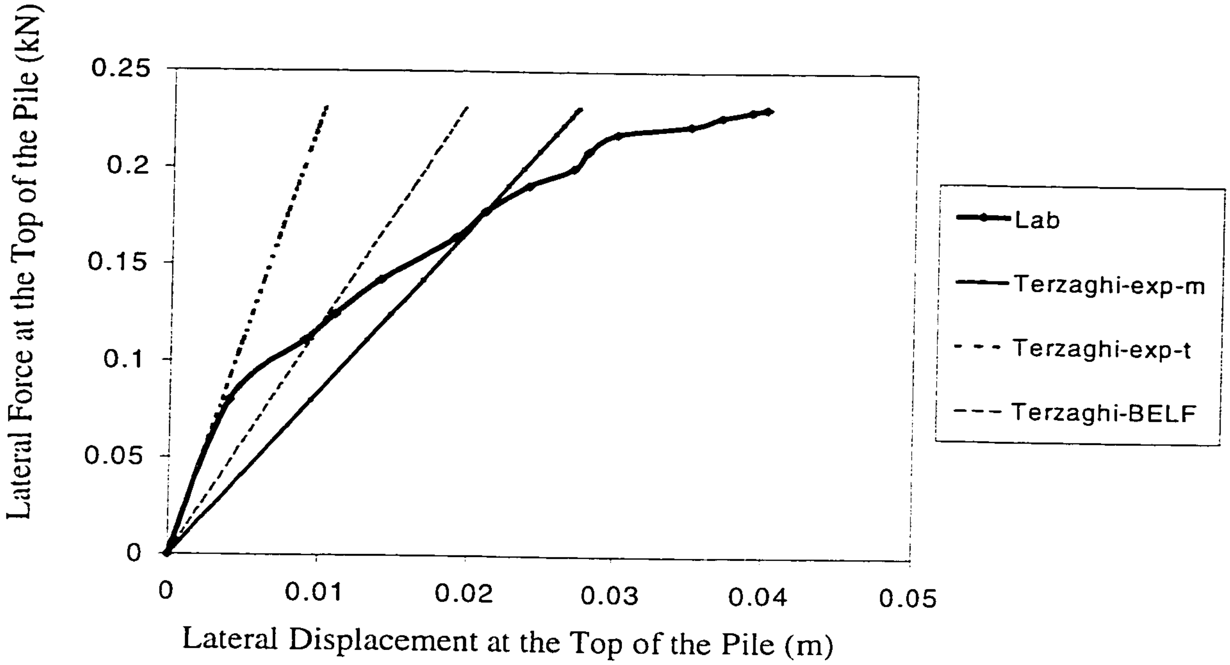


Figure B.25 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

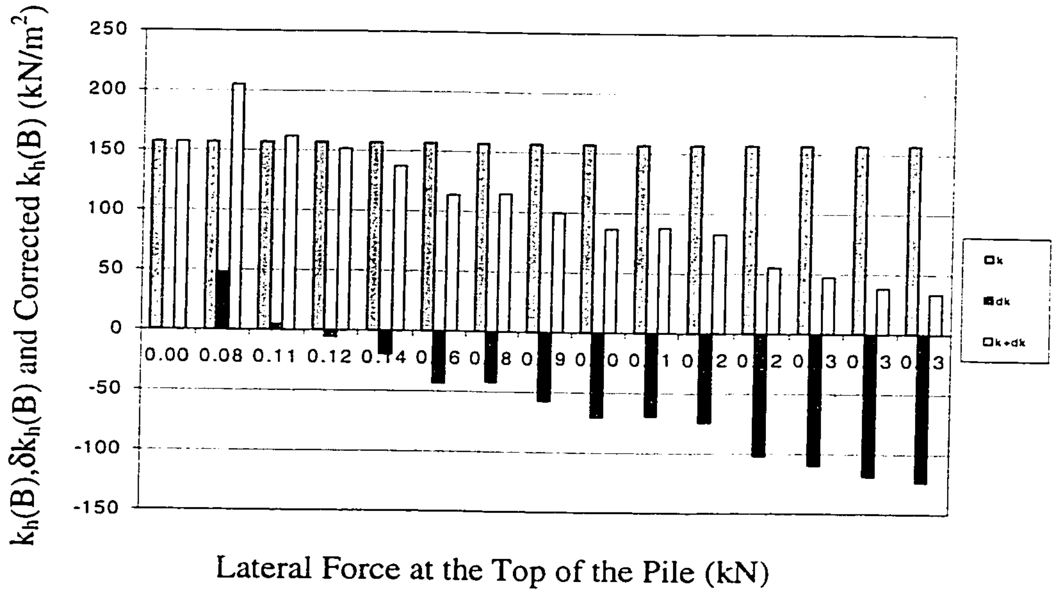


Figure B.26 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.6 Width of Pile = 38.1mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 165 kN/m^2

Table B.2.1.6 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.0801	0.006372	-0.00274	0.0035	-0.001512	0.00005103	30
22	0.0978	0.007788	-0.00334	0.005	-0.002788	0.00006237	45
26	0.11556	0.009204	-0.004	0.008	-0.001204	0.00007371	16
32	0.14234	0.01133	-0.00486	0.011	-0.00033	0.00009072	4
36	0.16013	0.01274	-0.00547	0.015	0.00226	0.00010206	-22
40	0.17793	0.014	-0.00608	0.017	0.00284	0.0001134	-25
43	0.19127	0.01499	-0.00654	0.019	0.00378	0.000122	-31
46	0.20462	0.0163	-0.007	0.021	0.0047	0.00013041	-36
48	0.2135	0.017	-0.0073	0.023	0.006	0.00013608	-44
50	0.22241	0.0177	-0.0076	0.024	0.0063	0.00014175	-44
51	0.22686	0.01805	-0.00775	0.026	0.008	0.0001446	-55
52	0.2313	0.01841	-0.0079	0.028	0.0096	0.00014742	-65
53	0.2357	0.01876	-0.00806	0.031	0.01224	0.0001503	-81
54	0.2402	0.01912	-0.00821	0.032	0.01288	0.0001531	-84
54.5	0.2424	0.0193	-0.008284	0.034	0.0147	0.0001545	-95
55	0.24465	0.1947	-0.00836	0.035	0.01553	0.000156	-100

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
195	5109	0.00345	0.005
210	5505	0.00485	0.015
181	4760	0.00788	0.012
169	4427	0.011	0.0
143	3750	0.0159	0.09
140	3674	0.0174	0.04
134	3518	0.0192	0.02
129	3386	0.0211	0.01
121	3174	0.0239	0.09
121	3165	0.0248	0.08
110	2879	0.0265	0.05
100	2622	0.0279	0.01
84	2194	0.0316	0.06
81	2123	0.0331	0.11
70	1834	0.0352	0.12

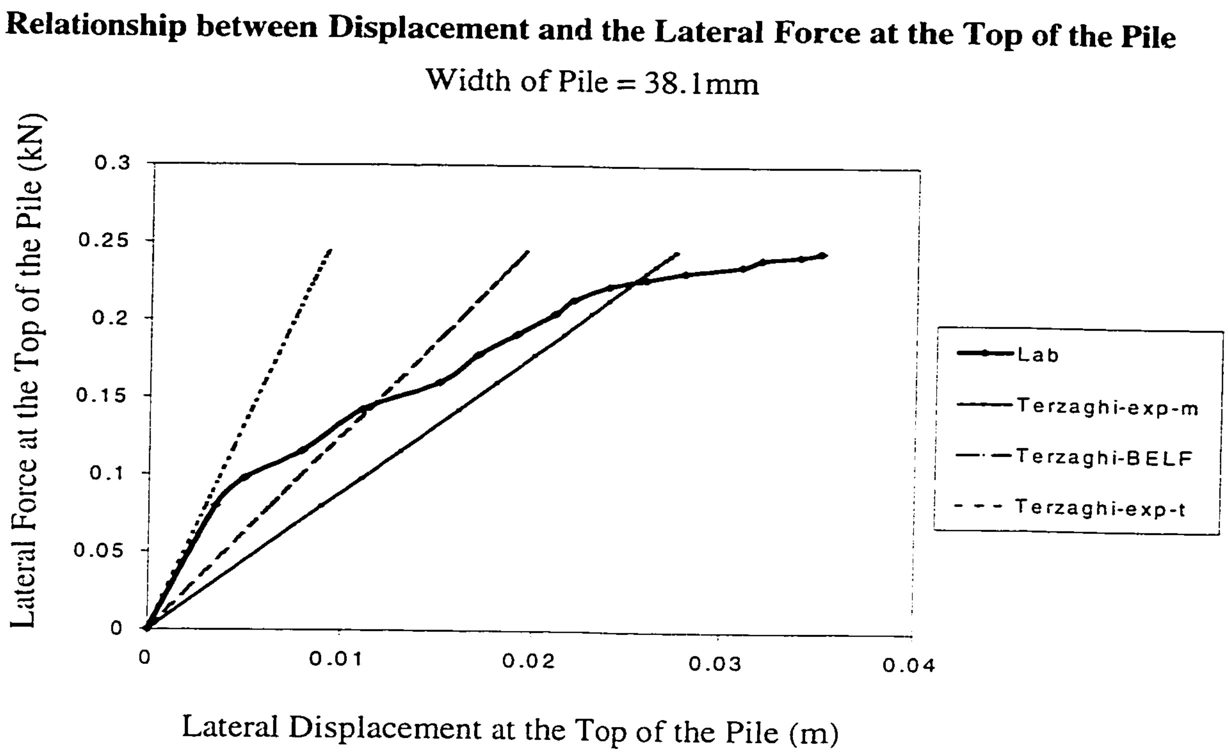


Figure B.27 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

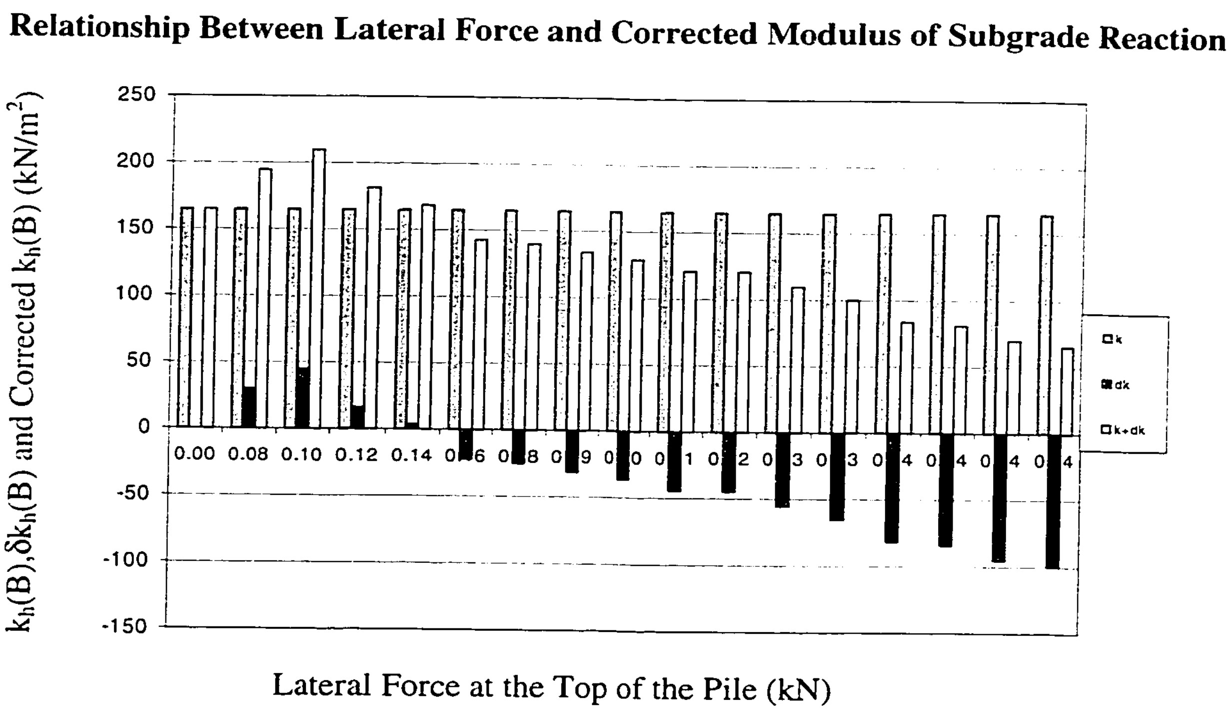


Figure B.28 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.7 Width of Pile = 44.45 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 220 kN/m^2

Table B.2.1.7 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.08	0.00482	-0.00203	0.003	-0.001824	0.0000293	62
25	0.1112	0.00670	-0.00282	0.006	-0.0007	0.0000408	17
30	0.1334	0.00804	-0.00338	0.009	0.00096	0.0000489	-20
33	0.1467	0.00884	-0.00372	0.01	0.001156	0.0000538	-21
37	0.1645	0.00992	-0.00417	0.012	0.002084	0.0000603	-35
45	0.2002	0.01206	-0.00507	0.017	0.00494	0.0000734	-67
47	0.2091	0.01260	-0.0053	0.019	0.006404	0.0000766	-84
50	0.2224	0.01340	-0.00564	0.021	0.0076	0.0000815	-93
55	0.2446	0.01474	-0.0062	0.024	0.00926	0.0000897	-103
56	0.2491	0.01501	-0.00632	0.026	0.010992	0.0000913	-120
57	0.25355	0.01528	-0.00643	0.028	0.012724	0.0000929	-137
58	0.258	0.01554	-0.00654	0.03	0.014456	0.0000945	-153
59	0.2624	0.01581	-0.00665	0.034	0.018188	0.0000962	-189
59.5	0.2646	0.01595	-0.00671	0.036	0.020054	0.0000970	-207
60	0.267	0.01608	-0.00677	0.037	0.02092	0.0000978	-214

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
282	6349	0.003	0
237	5337	0.0061	0.01
200	4509	0.0096	0.06
199	4467	0.011	0.1
185	4173	0.0123	0.03
153	3435	0.0172	0.02
136	3070	0.0196	0.06
127	2852	0.0211	0.01
117	2626	0.0248	0.08
100	2241	0.0269	0.09
83	1869	0.0288	0.08
67	1510	0.031	0.1
31	696	0.0345	0.05
13	298	0.0375	0.15
6	138	0.0382	0.12

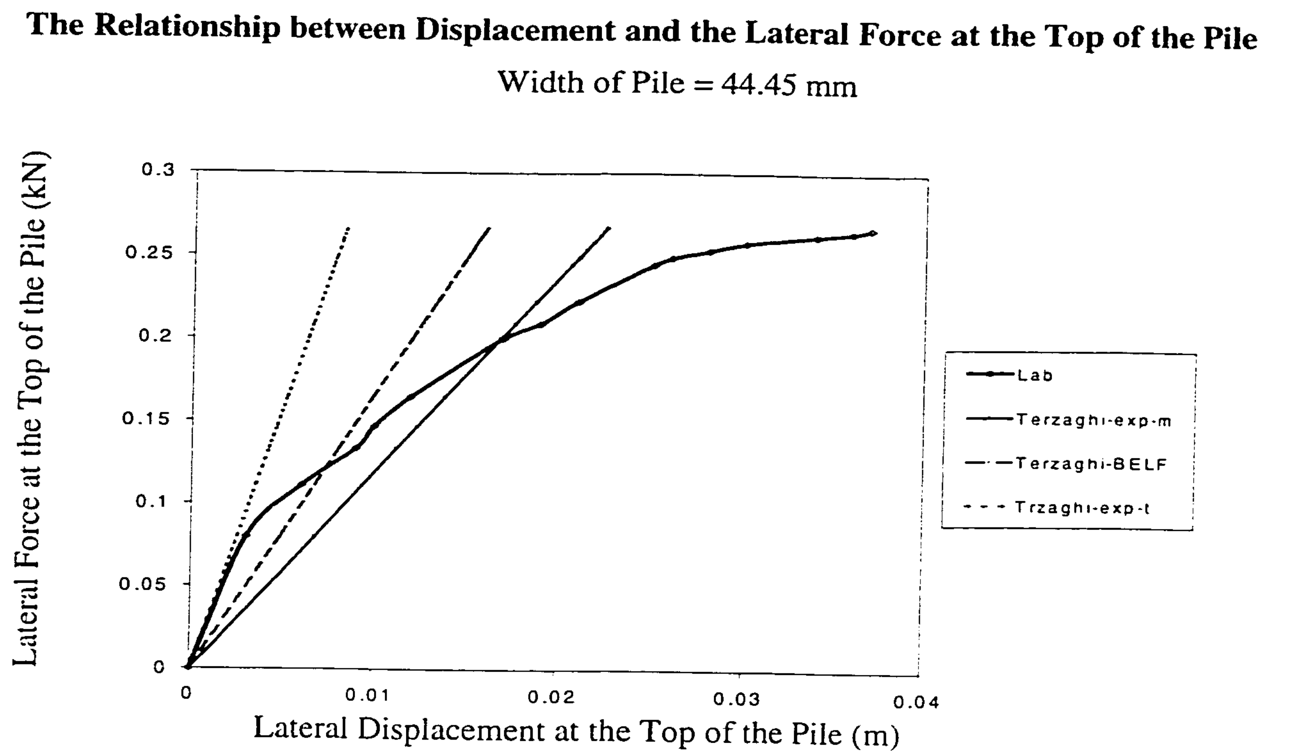


Figure B.29 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

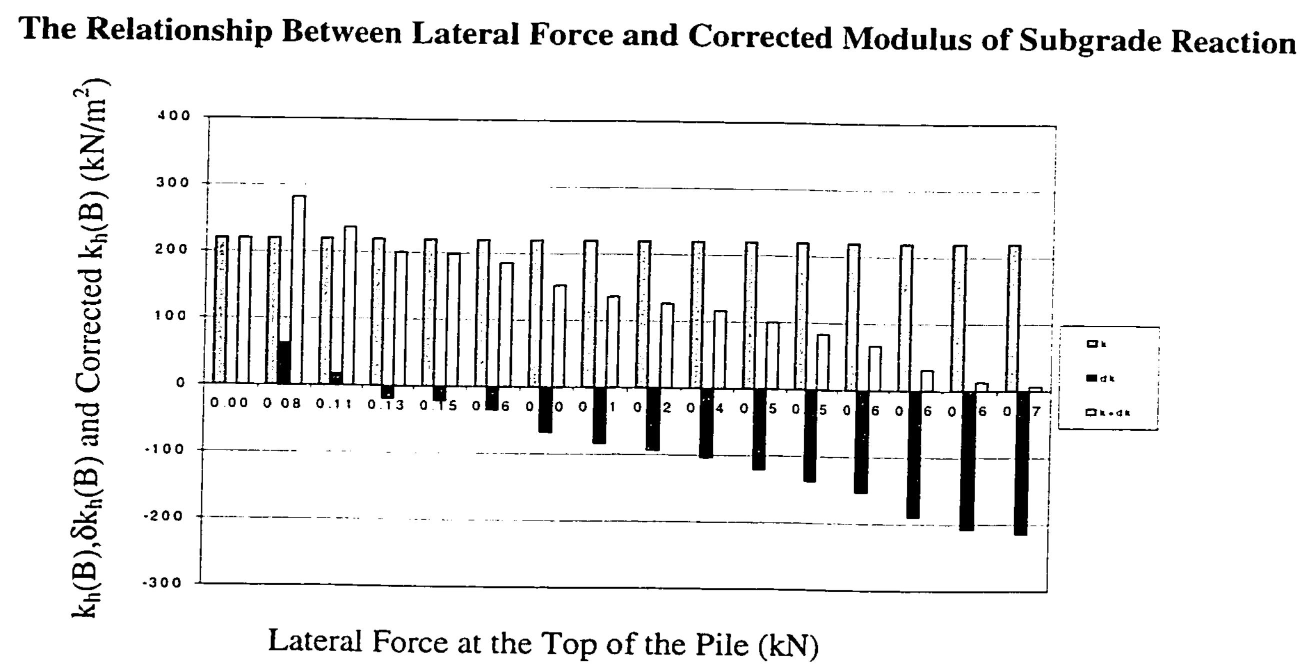


Figure B.30 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.1 Terzaghi's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.1.8 Width of Pile = 50.8 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 236 kN/m^2

Table B.2.1.8 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.08007	0.00450	-0.0018	0.002	-0.0025	0.0000256	98
25	0.11121	0.00625	-0.0025	0.004	-0.00225	0.0000356	63
27	0.12010	0.00675	-0.0027	0.005	-0.00175	0.0000384	46
30	0.13345	0.00750	-0.003	0.007	-0.0005	0.0000427	12
35	0.15569	0.00875	-0.0035	0.009	0.00025	0.0000498	-5
40	0.17793	0.01000	-0.004	0.012	0.002	0.0000569	-35
45	0.20017	0.01125	-0.0045	0.016	0.00475	0.0000640	-74
50	0.22241	0.01250	-0.005	0.019	0.0065	0.0000712	-91
55	0.24465	0.01375	-0.0055	0.023	0.00925	0.0000783	-118
57	0.25355	0.01425	-0.0057	0.026	0.01175	0.0000811	-145
58	0.25800	0.01450	-0.0058	0.028	0.0135	0.0000826	-164
59	0.26244	0.01475	-0.0059	0.029	0.01425	0.0000840	-170
60	0.26689	0.01500	-0.006	0.031	0.016	0.0000854	-187
61	0.27134	0.01525	-0.0061	0.034	0.01875	0.0000868	-216
61.5	0.27356	0.01538	-0.00615	0.035	0.019625	8.7533E-05	-224
62	0.27579	0.01550	-0.0062	0.036	0.0205	8.8245E-05	-232

- Calculation of Correct Values of Coefficient of Modulus of Subgrade Reaction and Lateral Displacement.

$k_{cor}(kN/m^2)$	$k_{cor}(kN/m^3)$	$v_{cor}(m)$	$\lvert Error\% \rvert$
334	6562	0.0021	0.01
299	5886	0.0042	0.02
282	5537	0.0055	0.05
248	4871	0.0074	0.04
231	4542	0.0093	0.03
201	3949	0.0129	0.09
162	3181	0.0167	0.07
145	2843	0.0198	0.08
118	2315	0.0241	0.11
91	1790	0.027	0.1
72	1422	0.0289	0.09
66	1300	0.0305	0.15
49	952	0.0321	0.11
20	390	0.035	0.1

The Relationship between Displacement and the Lateral Force at the Top of the Pile

Width of Pile = 50.8 mm

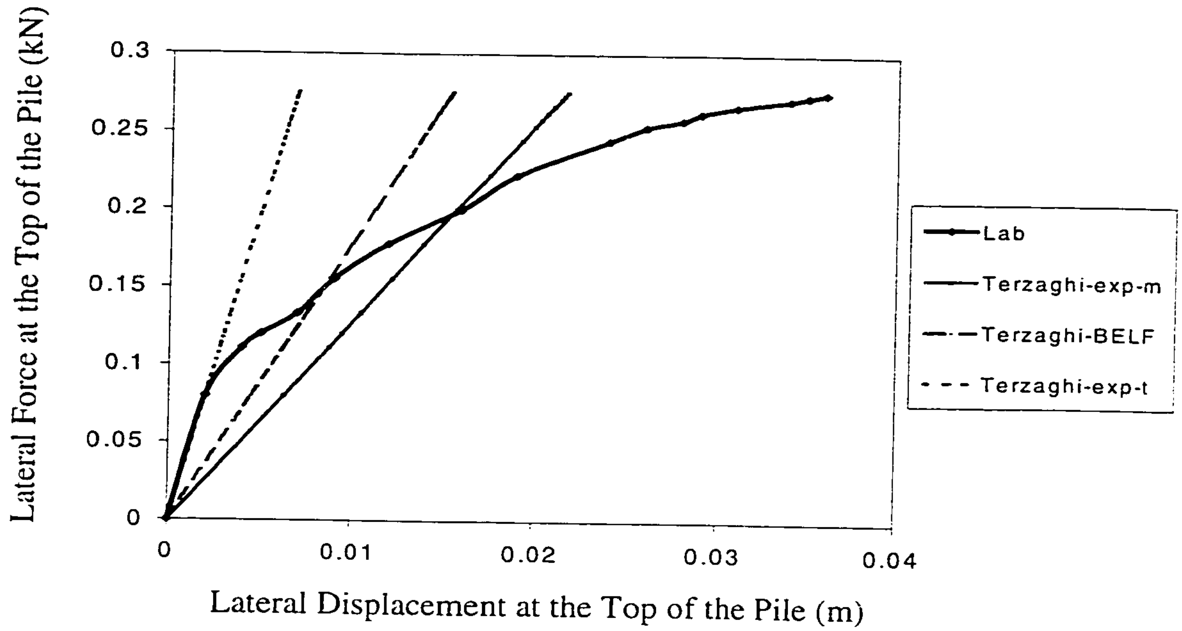


Figure B.31 The Relationship Between Displacement and the Lateral Force at the Top of the Pile.

The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction

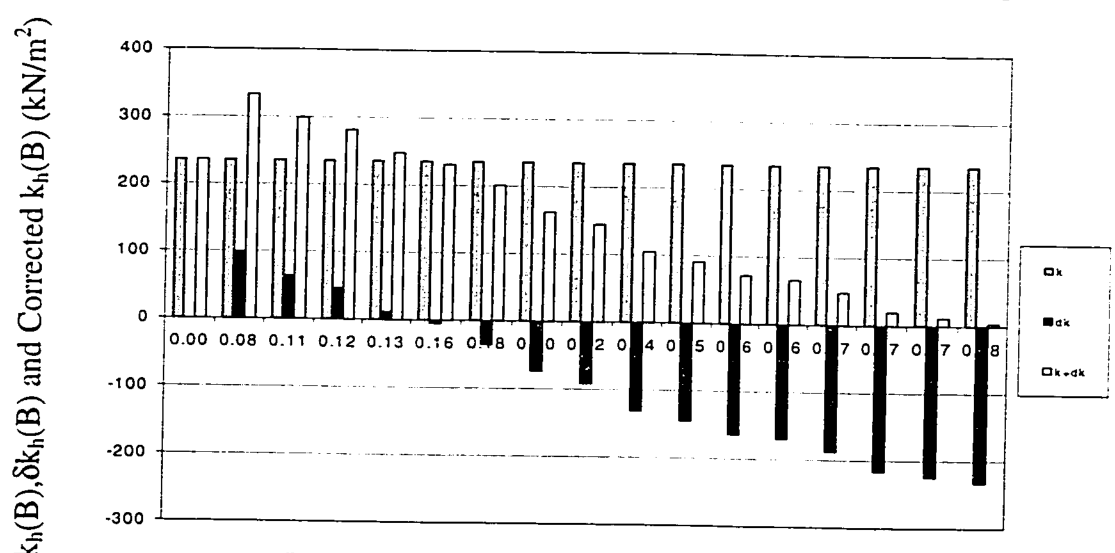


Figure B.32 The Relationship Between Lateral Force and Corrected Modulus of Subgrade Reaction.

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.2 Bowles' Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.2.1 Width of Pile = 6.35 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 12 kN/m^2

Table B.2.2.1 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv'zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
15	0.06672	0.0695	-0.03225	0.01	-0.0595	0.007251	8	20
20	0.08896	0.09266	-0.043	0.013	-0.07966	0.009668	8	20
25	0.1112	0.1158	-0.05375	0.020	-0.0958	0.012085	8	20
26	0.1156532	0.1205	-0.0559	0.027	-0.0935	0.01257	7	19
27	0.1201014	0.1251	-0.05805	0.034	-0.0911	0.01305	7	19
28	0.1245496	0.1297	-0.0602	0.037	-0.0927	0.013535	7	19
28.7	0.12766	0.133	-0.06171	0.038	-0.095	0.01387	7	19
29	0.129	0.1344	-0.06235	0.041	-0.0934	0.01402	7	19
29.2	0.13	0.1353	-0.06278	0.042	-0.0933	0.01412	7	19

B.2.2.2 Width of Pile = 12.7 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 24 kN/m^2

Table B.2.2.2 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
15	0.066723	0.03475	-0.01614	0.007	-0.02775	0.001813	15.	39
20	0.088964	0.04633	-0.02152	0.011	0.06672	0.03475	15	39
25	0.111205	0.0579	-0.0269	0.015	-0.0429	0.003022	14	38
27	0.1201	0.06254	-0.02905	0.019	-0.0435	0.003263	13	37
28	0.12455	0.06486	-0.03013	0.021	-0.04386	0.003384	13	37
29	0.129	0.0672	-0.0312	0.023	-0.0442	0.003505	13	37
30	0.1334	0.0695	-0.03228	0.025	-0.0445	0.003626	12	36
33	0.14679	0.07644	-0.03551	0.029	-0.04744	0.004	12	36
34	0.15124	0.07876	-0.03658	0.03	-0.04876	0.00411	12	36
35	0.15568	0.0811	-0.03766	0.031	-0.0501	0.00423	12	36
36	0.160135	0.0834	-0.03874	0.036	-0.0474	0.004351	11	35
36.5	0.162359	0.08455	-0.0393	0.039	-0.04555	0.004412	10	35
36.8	0.16369	0.08525	-0.0396	0.040	-0.04525	0.004448	10	34
37	0.16458	0.0857	-0.03981	0.041	-0.0447	0.00447	10	34

B.2.2.3 Width of Pile = 19.05 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 36 kN/m^2

Table B.2.2.3 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
15	0.06672	0.02317	-0.01076	0.005	-0.01817	0.000805	23	59
17	0.07562	0.00263	-0.01219	0.006	0.003375	0.000912	-4	32
22	0.09786	0.03398	-0.01578	0.01	-0.02398	0.001181	20	16
27	0.12010	0.04170	-0.01936	0.014	-0.0277	0.001449	19	17
30	0.13345	0.04633	-0.02152	0.017	-0.02933	0.00161	18	18
33	0.14679	0.05097	-0.02367	0.021	-0.02997	0.001771	17	19
36	0.16014	0.05560	-0.02582	0.025	-0.0306	0.001932	16	20
37	0.16458	0.05714	-0.02654	0.026	-0.03114	0.001986	16	20
39	0.17348	0.06023	-0.02797	0.029	-0.03123	0.002093	15	21
39.5	0.17570	0.06100	-0.02833	0.031	-0.03	0.00212	15	21
40	0.17793	0.06178	-0.02869	0.033	-0.02878	0.002147	13	21
40.8	0.18149	0.06301	-0.02926	0.035	-0.02801	0.00219	13	21
41	0.18238	0.06332	-0.02941	0.036	-0.02732	0.0022	12	22

B.2.2.4 Width of Pile = 25.4 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 48 kN/m^2

Table B.2.2.4 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
15	0.06672	0.01737	-0.00807	0.004	-0.01337	0.000453	30	78
18	0.08007	0.02084	-0.00968	0.005	-0.01584	0.000544	29	77
23	0.10231	0.02663	-0.01237	0.009	-0.01763	0.000695	25	73
28	0.12455	0.03242	-0.01506	0.014	-0.01842	0.000846	22	70
30	0.13345	0.03474	-0.01614	0.016	-0.01874	0.000906	21	69
33	0.14679	0.03821	-0.01775	0.019	-0.01921	0.000997	19	67
36	0.16014	0.04169	-0.01937	0.022	-0.01969	0.001087	18	66
40	0.17793	0.04632	-0.02152	0.026	-0.02032	0.001208	17	65
42	0.18682	0.04864	-0.0226	0.028	-0.02064	0.001268	16	64
43	0.19127	0.04979	-0.02313	0.029	-0.02079	0.001299	16	64
44	0.19572	0.05095	-0.02367	0.033	-0.01795	0.001329	14	62
44.5	0.19794	0.05153	-0.02394	0.036	-0.01553	0.001344	12	60
45	0.20017	0.05211	-0.02421	0.037	-0.01511	0.001359	11	59

B.2.2.5 Width of Pile = 31.75 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 60 kN/m^2

Table B.2.2.5 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
18	0.08007	0.01667	-0.00774	0.004	-0.01267	0.000347	36	96
25	0.11121	0.02315	-0.01075	0.009	-0.01415	0.000483	29	89
28	0.12455	0.02593	-0.01204	0.011	-0.01493	0.00054	28	88
32	0.14234	0.02963	-0.01376	0.014	-0.01563	0.000618	25	85
37	0.16458	0.03426	-0.01591	0.019	-0.01526	0.000714	21	81
40	0.17793	0.03704	-0.0172	0.021	-0.01604	0.000772	21	81
43	0.19127	0.03982	-0.01849	0.024	-0.01582	0.00083	19	79
45	0.20017	0.04167	-0.01935	0.027	-0.01467	0.000869	17	77
47	0.20907	0.04352	-0.02021	0.028	-0.01552	0.000907	17	77
49	0.21796	0.04537	-0.02107	0.03	-0.01537	0.000946	16	76
50	0.22241	0.04630	-0.0215	0.035	-0.0113	0.000965	12	72
51	0.22686	0.04723	-0.02193	0.037	-0.01023	0.000984	10	70
51.7	0.22997	0.04787	-0.02223	0.039	-0.00887	0.000998	9	69
52	0.23131	0.04815	-0.02236	0.04	-0.00815	0.001004	8	68

B.2.2.6 Width of Pile = 38.1mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 72 kN/m^2

Table B.2.2.6 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
18	0.08007	0.01390	-0.00644	0.0035	-0.0104	0.000241	43	115
22	0.09786	0.01698	-0.00788	0.005	-0.01198	0.000295	41	113
26	0.11565	0.02007	-0.00931	0.008	-0.01207	0.000348	35	107
32	0.14234	0.02470	-0.01146	0.011	-0.0137	0.000429	32	104
36	0.16014	0.02779	-0.01289	0.015	-0.01279	0.000482	27	99
40	0.17793	0.03088	-0.01432	0.017	-0.01388	0.000536	26	98
43	0.19127	0.03320	-0.01539	0.019	-0.0142	0.000576	25	97
46	0.20462	0.03551	-0.01647	0.021	-0.01451	0.000616	24	96
48	0.21351	0.03706	-0.01718	0.023	-0.01406	0.000643	22	94
50	0.22241	0.03860	-0.0179	0.024	-0.0146	0.00067	22	94
51	0.22686	0.03937	-0.01826	0.026	-0.01337	0.000683	20	92
52	0.23131	0.04014	-0.01862	0.028	-0.01214	0.000697	17	89
53	0.23575	0.04092	-0.01897	0.031	-0.00992	0.00071	14	86
54	0.24020	0.04169	-0.01933	0.032	-0.00969	0.000724	13	85
54.5	0.24243	0.042074	-0.01951	0.034	-0.00807	0.00073	11	83
55	0.24465	0.04246	-0.01969	0.035	-0.00746	0.000737	10	82

B.2.2.7 Width of Pile =44.45 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 84 kN/m^2

Table B.2.2.7 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
18	0.08007	0.01192	-0.00558	0.003	-0.00892	0.000177	50	134
25	0.11121	0.01655	-0.00775	0.006	-0.01055	0.000246	43	127
30	0.13345	0.01986	-0.0093	0.009	-0.01086	0.000296	37	121
33	0.14679	0.02185	-0.01023	0.01	-0.01185	0.000325	36	120
37	0.16458	0.02449	-0.01147	0.012	-0.01249	0.000364	34	118
45	0.20017	0.02979	-0.01395	0.017	-0.01279	0.000443	29	113
47	0.20907	0.03111	-0.01457	0.019	-0.01211	0.000463	26	110
50	0.22241	0.03310	-0.0155	0.021	-0.0121	0.000493	25	109
55	0.24465	0.03641	-0.01705	0.025	-0.01141	0.000542	21	105
56	0.24910	0.03707	-0.01736	0.026	-0.01107	0.000552	20	104
57	0.25355	0.03773	-0.01767	0.028	-0.00973	0.000561	17	101
58	0.25800	0.03840	-0.01798	0.03	-0.0084	0.000571	15	98
59	0.26244	0.03906	-0.01829	0.034	-0.00506	0.000581	9	92
59.5	0.26467	0.03939	-0.01845	0.036	-0.00339	0.000586	6	89
60	0.26689	0.03972	-0.0186	0.037	-0.00272	0.000591	5	88

B.2.2.8 Width of Pile = 50.8 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 96 kN/m^2

Table B.2.2.8 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$	$k_c(kN/m^2)$
18	0.08007	0.01046	-0.00486	0.002	-0.00846	0.000137	62	158
25	0.11121	0.01453	-0.00675	0.004	-0.01053	0.00019	55	151
27	0.12010	0.01569	-0.00729	0.005	-0.01069	0.000205	52	148
30	0.13345	0.01743	-0.0081	0.007	-0.01043	0.000228	46	142
35	0.15569	0.02034	-0.00945	0.009	-0.01134	0.000266	43	139
40	0.17793	0.02324	-0.0108	0.012	-0.01124	0.000304	37	133
45	0.20017	0.02615	-0.01215	0.016	-0.01014	0.000342	30	126
50	0.22241	0.02905	-0.0135	0.019	-0.01005	0.00038	26	122
55	0.24465	0.03196	-0.01485	0.024	-0.00796	0.000418	19	115
57	0.25355	0.03312	-0.01539	0.026	-0.00712	0.000433	16	112
58	0.25800	0.03370	-0.01566	0.028	-0.0057	0.000441	13	109
59	0.26244	0.03428	-0.01593	0.029	-0.00528	0.000448	12	108
60	0.26689	0.03486	-0.0162	0.031	-0.00386	0.000456	8	104
61	0.27134	0.03544	-0.01647	0.034	-0.00144	0.000464	3	99
61.5	0.27356	0.035732	-0.01661	0.035	-0.00073	0.000467	2	98
62	0.27579	0.036022	-0.01674	0.036	-0.00002	0.000471	0	96

B.2 Analysis of Piles Embedded in Clayey Soil Subjected to Lateral Force

B.2.3 Vesic's Method Used for Determination of Initial Modulus of Subgrade Reaction.

B.2.3.1 Width of Pile = 6.35 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 429 kN/m^2

Table B.2.3.1 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.00340	-0.00028	0.01	0.006604	0.0000213	-310
20	0.08896	0.00453	-0.00038	0.013	0.008472	0.0000284	-298
25	0.11121	0.00566	-0.00047	0.02	0.01434	0.0000355	-404
26	0.11565	0.00589	-0.00049	0.027	0.021114	0.0000369	-572
27	0.12010	0.00611	-0.00051	0.034	0.027887	0.0000383	-727
28	0.12455	0.00634	-0.00053	0.037	0.030661	0.0000398	-771
28.7	0.12766	0.00650	-0.00054	0.038	0.031502	0.0000408	-773
29	0.12900	0.00657	-0.00055	0.041	0.034434	0.0000412	-836
29.2	0.12989	0.00661	-0.00055	0.042	0.035389	0.0000415	-853

B.2.3.2 Width of Pile = 12.7 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 510 kN/m^2

Table B.2.3.2 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 z dz$	$\delta k(kN/m^2)$
15	0.06672	0.00245	-0.00039	0.007	0.004549	0.0000104	-437
20	0.08896	0.00327	-0.00052	0.011	0.007732	0.0000139	-557
25	0.11121	0.00409	-0.00065	0.015	0.010915	0.0000173	-630
27	0.12010	0.00441	-0.0007	0.019	0.014588	0.0000187	-779
28	0.12455	0.00458	-0.00073	0.021	0.016425	0.0000194	-846
29	0.12900	0.00474	-0.00075	0.023	0.018261	0.0000201	-908
30	0.13345	0.00490	-0.00078	0.025	0.020098	0.0000208	-966
33	0.14679	0.00539	-0.00086	0.029	0.023608	0.0000229	-1032
34	0.15124	0.00556	-0.00088	0.03	0.024444	0.0000236	-1037
35	0.15569	0.00572	-0.00091	0.031	0.025281	0.0000243	-1042
36	0.16014	0.00588	-0.00094	0.036	0.030118	0.0000250	-1207
36.5	0.16236	0.00596	-0.00095	0.039	0.033036	0.0000253	-1305
36.8	0.16369	0.00601	-0.00096	0.04	0.033987	0.0000255	-1332
37	0.16458	0.00605	-0.00096	0.041	0.034954	0.0000257	-1362

B.2.3.3 Width of Pile = 19.05 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 565 kN/m^2

Table B.2.3.3 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.00205	-0.00042	0.005	0.002951	0.0000071	-413
17	0.07562	0.00232	-0.00048	0.006	0.003678	0.0000081	-454
22	0.09786	0.00301	-0.00062	0.01	0.006995	0.0000105	-667
27	0.12010	0.00369	-0.00076	0.014	0.010312	0.0000129	-801
30	0.13345	0.00410	-0.00085	0.017	0.012902	0.0000143	-902
33	0.14679	0.00451	-0.00093	0.021	0.016492	0.0000157	-1048
36	0.16014	0.00492	-0.00102	0.025	0.020082	0.0000172	-1170
37	0.16458	0.00505	-0.00104	0.026	0.020946	0.0000176	-1188
39	0.17348	0.00533	-0.0011	0.029	0.023673	0.0000186	-1273
39.5	0.17570	0.00540	-0.00111	0.031	0.025604	0.0000188	-1360
40	0.17793	0.00546	-0.00113	0.033	0.027536	0.0000191	-1444
40.8	0.18149	0.00557	-0.00115	0.035	0.029427	0.0000194	-1513
41	0.18238	0.00560	-0.00116	0.036	0.030399	0.0000195	-1555

B.2.3.4 Width of Pile = 25.4 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 607 kN/m^2

Table.B.2.3.4 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
15	0.06672	0.00180	-0.00044	0.004	0.0022	0.0000053	-415
18	0.08007	0.00216	-0.00052	0.005	0.00284	0.0000064	-446
23	0.10231	0.00276	-0.00067	0.009	0.00624	0.0000081	-768
28	0.12455	0.00336	-0.00081	0.014	0.01064	0.0000099	-1075
30	0.13345	0.00360	-0.00087	0.016	0.0124	0.0000106	-1170
33	0.14679	0.00396	-0.00096	0.019	0.01504	0.0000117	-1290
36	0.16014	0.00432	-0.00104	0.022	0.01768	0.0000127	-1390
40	0.17793	0.00480	-0.00116	0.026	0.0212	0.0000141	-1500
42	0.18682	0.00504	-0.00122	0.028	0.02296	0.0000148	-1547
43	0.19127	0.00516	-0.00125	0.029	0.02384	0.0000152	-1569
44	0.19572	0.00528	-0.00128	0.033	0.02772	0.0000155	-1783
44.5	0.19794	0.00534	-0.00129	0.036	0.03066	0.0000157	-1950
45	0.20017	0.00540	-0.00131	0.037	0.0316	0.0000159	-1987

B.2.3.5 Width of Pile = 31.75 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 642 kN/m^2

Table B.2.3.5 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^{I} zdz$	$\delta k(kN/m^2)$
18	0.08007	0.00198	-0.00053	0.004	0.00202	0.0000052	-387
25	0.11121	0.00275	-0.00074	0.009	0.00625	0.0000073	-862
28	0.12455	0.00308	-0.00082	0.011	0.00792	0.0000081	-975
32	0.14234	0.00352	-0.00094	0.014	0.01048	0.0000093	-1129
37	0.16458	0.00407	-0.00109	0.019	0.01493	0.0000107	-1391
40	0.17793	0.00440	-0.00118	0.021	0.0166	0.0000116	-1431
43	0.19127	0.00473	-0.00126	0.024	0.01927	0.0000125	-1545
45	0.20017	0.00495	-0.00132	0.027	0.02205	0.0000131	-1690
47	0.20907	0.00517	-0.00138	0.028	0.02283	0.0000136	-1675
49	0.21796	0.00539	-0.00144	0.03	0.02461	0.0000142	-1732
50	0.22241	0.00550	-0.00147	0.035	0.0295	0.0000145	-2034
51	0.22686	0.00561	-0.0015	0.037	0.03139	0.0000148	-2122
51.7	0.22997	0.00569	-0.00152	0.039	0.033313	0.0000150	-2222
52	0.23131	0.00572	-0.00153	0.04	0.03428	0.0000151	-2273

B.2.3.6 Width of Pile = 38.1mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 672 kN/m^2

Table B.2.3.6 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 z dz$	$\delta k(kN/m^2)$
18	0.08007	0.00185	-0.00053	0.0035	0.001653	0.0000045	-367
22	0.09786	0.00226	-0.00064	0.005	0.002743	0.0000055	-499
26	0.11565	0.00267	-0.00076	0.008	0.005332	0.0000065	-820
32	0.14234	0.00328	-0.00094	0.011	0.007717	0.000008	-965
36	0.16014	0.00369	-0.00105	0.015	0.011306	0.000009	-1256
40	0.17793	0.00410	-0.00117	0.017	0.012896	0.00001	-1290
43	0.19127	0.00441	-0.00126	0.019	0.014588	0.00001075	-1357
46	0.20462	0.00472	-0.00135	0.021	0.01628	0.0000115	-1416
48	0.21351	0.00492	-0.00141	0.023	0.018075	0.000012	-1506
50	0.22241	0.00513	-0.00147	0.024	0.01887	0.0000125	-1510
51	0.22686	0.00523	-0.00149	0.026	0.020767	0.00001275	-1629
52	0.23131	0.00534	-0.00152	0.028	0.022665	0.000013	-1743
53	0.23575	0.00544	-0.00155	0.031	0.025562	0.00001325	-1929
54	0.24020	0.00554	-0.00158	0.032	0.02646	0.0000135	-1960
54.5	0.24243	0.005592	-0.0016	0.034	0.028408	1.3625E-05	-2085
55	0.24465	0.005643	-0.00161	0.035	0.029357	0.00001375	-2135

B.2.3.7 Width of Pile = 44.45 mm and the length of the model = 400 mm - $k_h(B)$ (initial) = 698 kN/m^2

Table B.2.3.7 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load(lb)$	$Load(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.08007	0.00174	-0.00053	0.003	0.001258	3.96E-06	-317
25	0.11121	0.00242	-0.00073	0.006	0.00358	5.5E-06	-651
30	0.13345	0.00290	-0.00088	0.009	0.006096	6.6E-06	-924
33	0.14679	0.00319	-0.00096	0.01	0.006806	7.26E-06	-937
37	0.16458	0.00358	-0.00108	0.012	0.008418	8.14E-06	-1034
45	0.20017	0.00436	-0.00131	0.017	0.012644	9.9E-06	-1277
47	0.20907	0.00455	-0.00137	0.019	0.01445	1.03E-05	-1397
50	0.22241	0.00484	-0.00146	0.021	0.01616	0.000011	-1469
55	0.24465	0.00532	-0.00161	0.025	0.019676	1.21E-05	-1626
56	0.24910	0.00542	-0.00164	0.026	0.020579	1.23E-05	-1670
57	0.25355	0.00552	-0.00166	0.028	0.022482	1.25E-05	-1793
58	0.25800	0.00561	-0.00169	0.03	0.024386	1.28E-05	-1911
59	0.26244	0.00571	-0.00172	0.034	0.028289	1.3E-05	-2179
59.5	0.26467	0.00576	-0.00174	0.036	0.03024	1.31E-05	-2310
60	0.26689	0.005808	-0.00175	0.037	0.031192	1.32E-05	-2363

B.2.3.8 Width of Pile = 50.8mm and the length of the model = 400 mm - $k_h(B)$ (initial)= 727 kN/m^2

Table B.2.3.8 Calculation of Variation of Modulus of Subgrade Reaction by Sensitivity Analysis

$Load\,(lb)$	$Load\,(kN)$	$v_{T(B)}(m)$	$v_{D(B)}(m)$	$v_{Lab}(m)$	$\delta v(m)$	$\int_0^l vv^1 zdz$	$\delta k(kN/m^2)$
18	0.08007	0.00167	-0.00053	0.01	0.00833	0.0000036	-2288
25	0.11121	0.00232	-0.00074	0.013	0.01068	0.0000051	-2112
27	0.12010	0.00251	-0.00079	0.02	0.017494	0.0000055	-3204
30	0.13345	0.00278	-0.00088	0.027	0.024216	0.0000061	-3991
35	0.15569	0.00325	-0.00103	0.034	0.030752	0.0000071	-4344
40	0.17793	0.00371	-0.00118	0.037	0.033288	0.0000081	-4115
45	0.20017	0.00418	-0.00132	0.038	0.033824	0.0000091	-3717
50	0.22241	0.00464	-0.00147	0.041	0.03636	0.0000101	-3596
55	0.24465	0.00510	-0.00162	0.042	0.036896	0.0000111	-3317
57	0.25355	0.00529	-0.00168	0.031	0.02571	0.0000115	-2230
58	0.25800	0.00538	-0.00171	0.036	0.030618	0.0000117	-2610
59	0.26244	0.00548	-0.00173	0.039	0.033525	0.0000119	-2809
60	0.26689	0.00557	-0.00176	0.04	0.034432	0.0000121	-2837
61	0.27134	0.00566	-0.00179	0.041	0.035339	0.0000123	-2864
61.5	0.27356	0.005707	-0.00181	0.034	0.028293	1.2438E-05	-2275
62	0.27579	0.005754	-0.00182	0.035	0.029246	1.2539E-05	-2332

Appendix C

THE RELATIONSHIP BETWEEN THE WIDTH OF THE PILE EMBEDDED IN HOMOGEOUS SOIL AND THE CONSTANT OF HORIZONTAL SUBGRADE REACTION

C.1 The Relationship Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction

C.1.1 $q = 1.5\ kN/m$

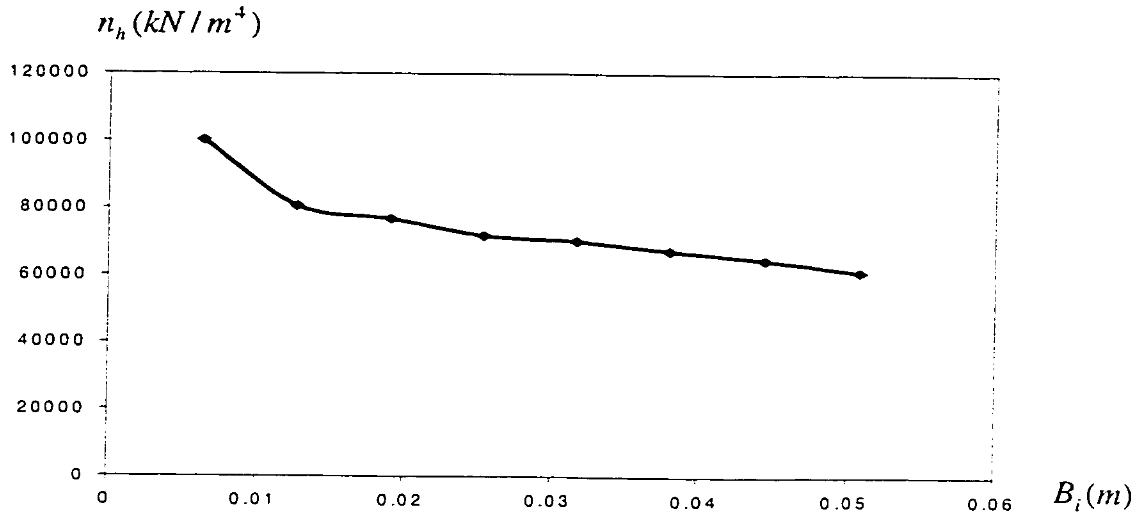


Figure C.1 The Relation Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction.

C.1.2 $q = 2.0\ kN/m$

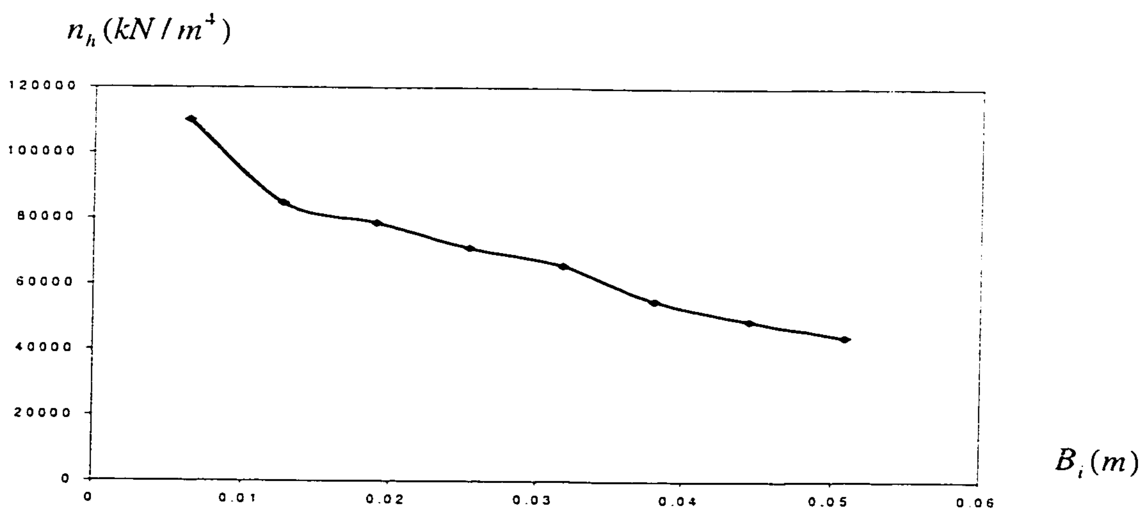


Figure C.2 The Relation Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction.

C.1 The Relationship Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction

C.1.3 $q = 2.5\ kN/m$

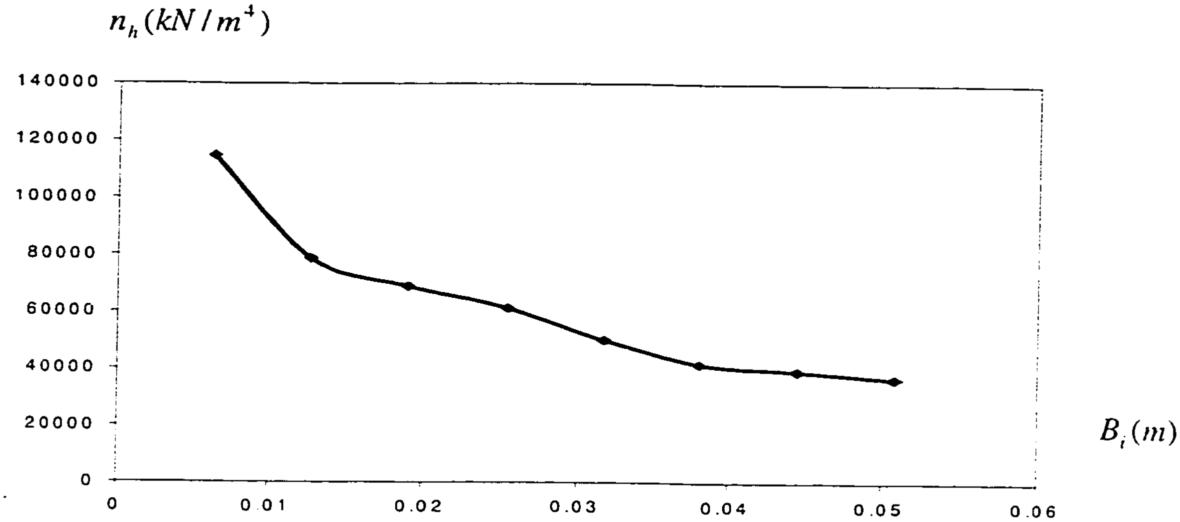


Figure C.3 The Relation Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction .

C.1.4 $q = 3.0\ kN/m$

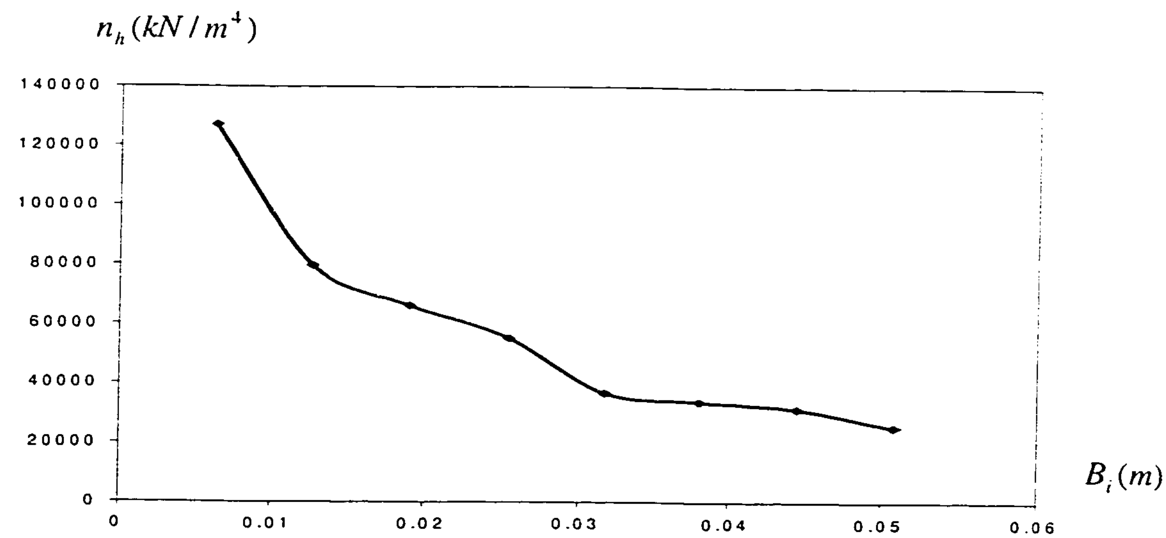


Figure C.4 The Relation Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction .

C.2 The Relationship Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction for all Values of q

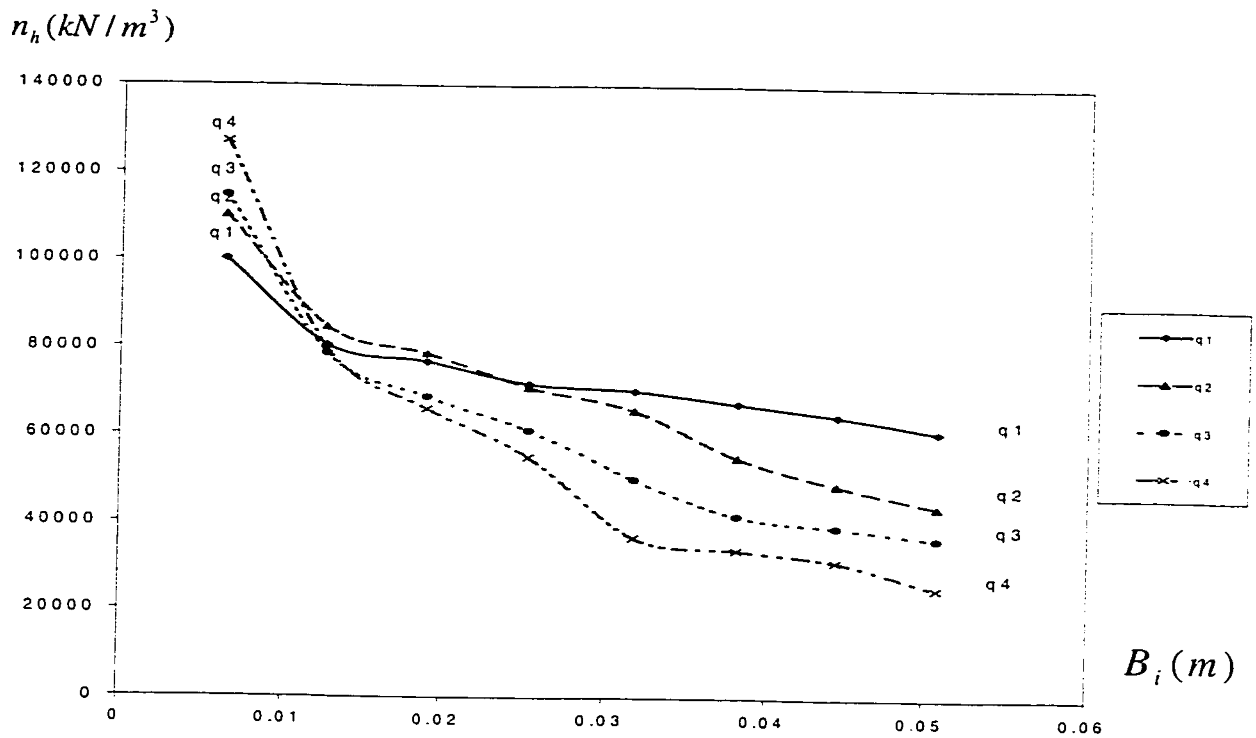


Figure C.5 The Relationship Between the Width of the Pile Embedded in Sandy Soil and the Constant of Horizontal Subgrade Reaction for all Values of q .

C.3 The Relationship Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction

C.3.1 $q = 3.5\ kN/m$

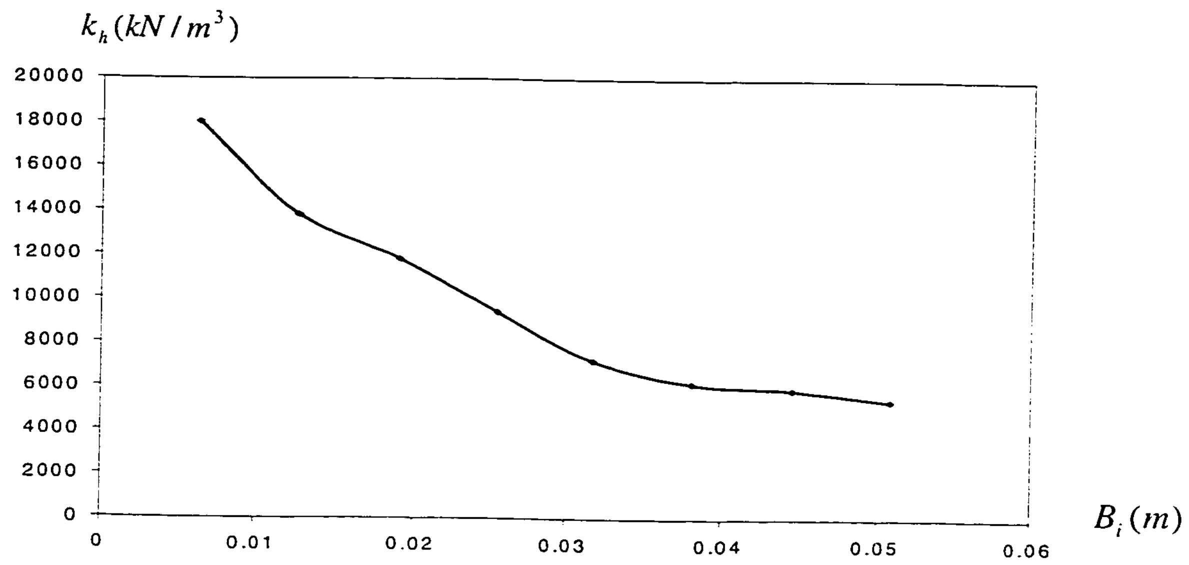


Figure C.6 The Relation Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction .

C.3.1 $q = 4.0\ kN/m$

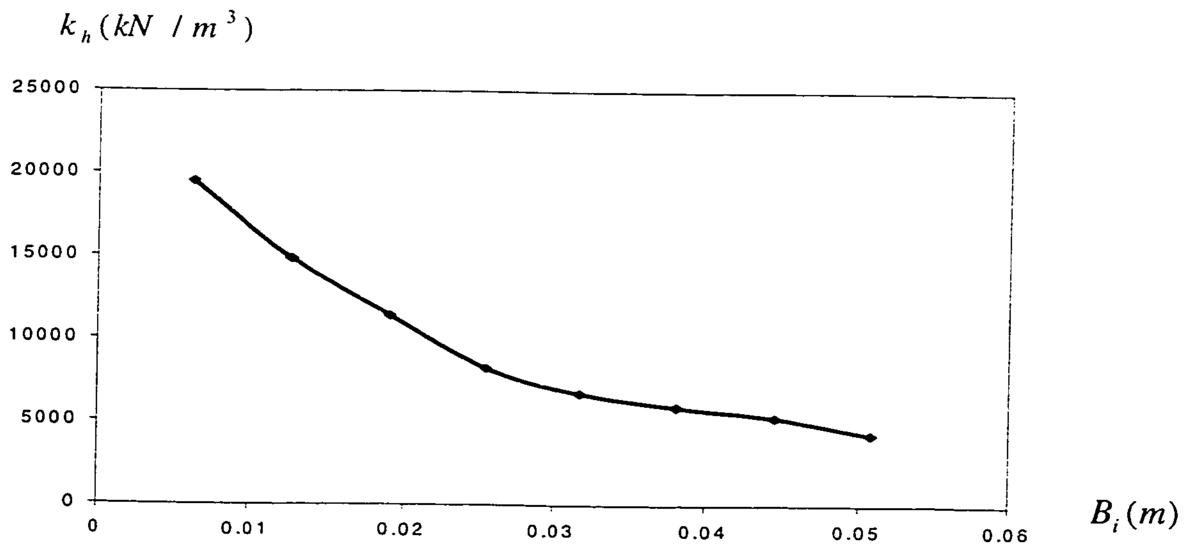


Figure C.7 The Relation Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction .

C.3 The Relationship Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction

C.3.3 $q = 4.5\ kN/m$

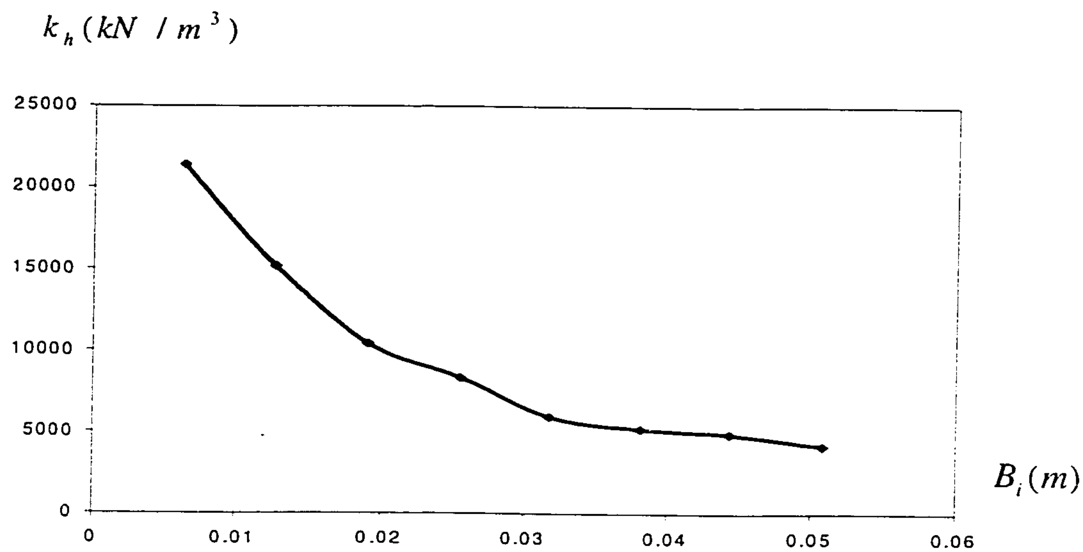


Figure C.8 The Relation Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction .

C.3.4 $q = 5.0\ kN/m$

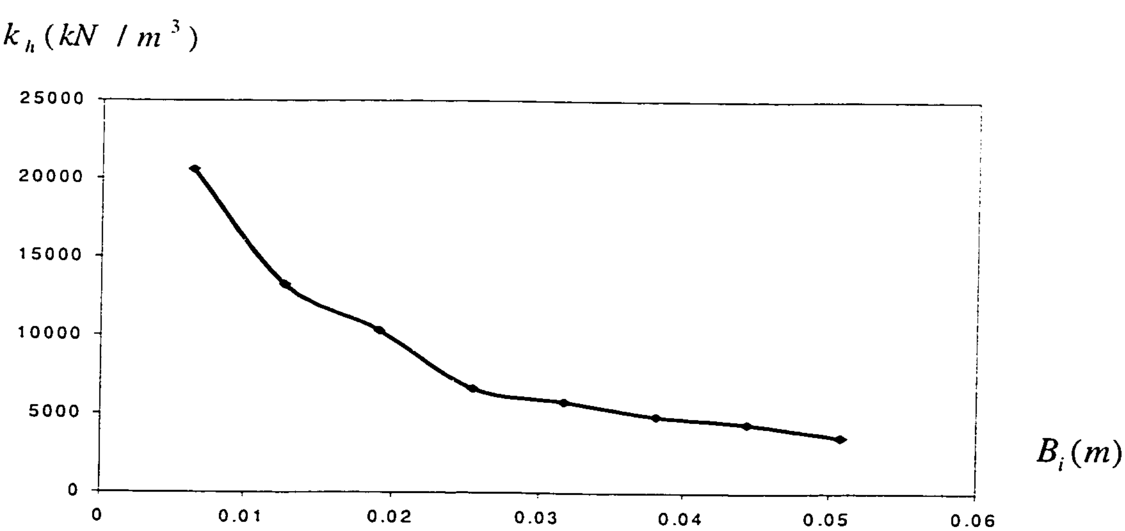


Figure C.9 The Relation Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction.

C.4 The Relationship Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction for all Values of q

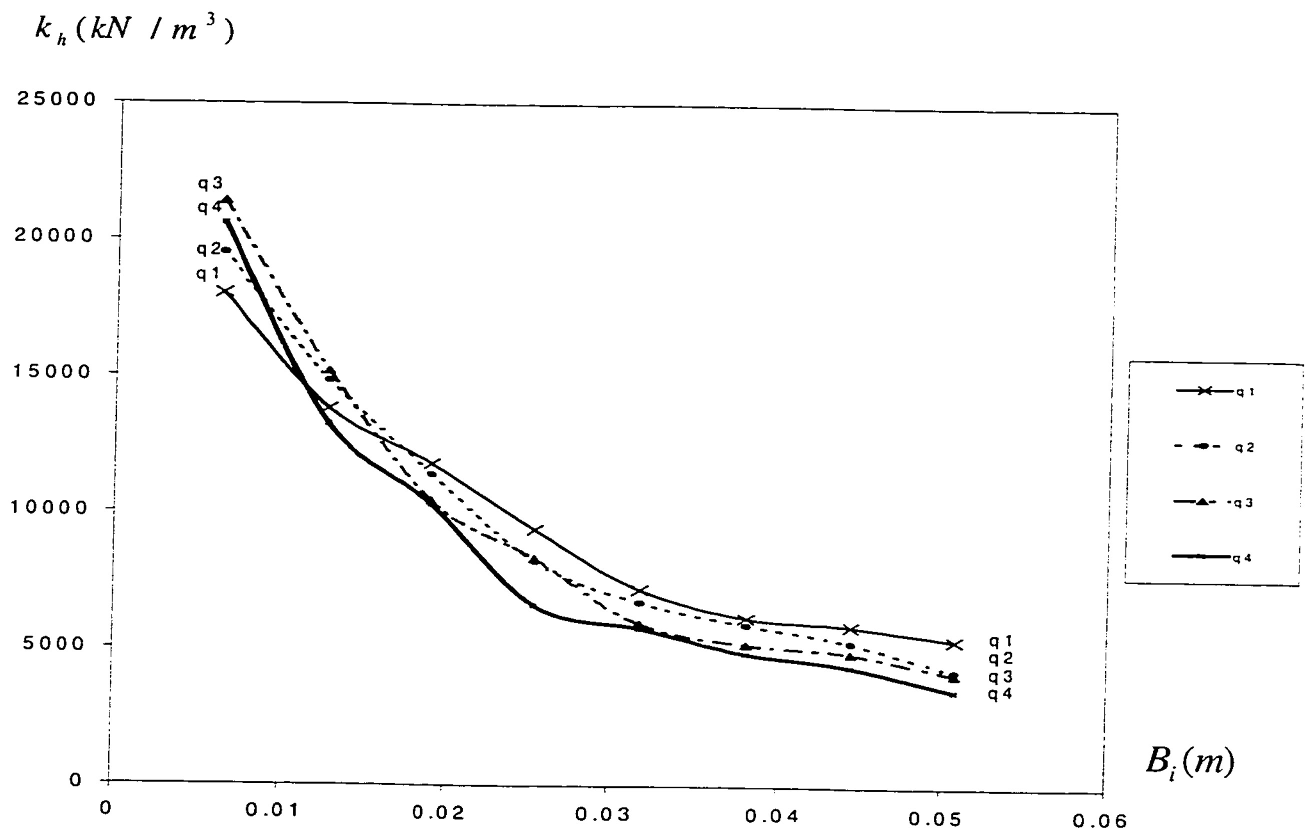


Figure C.10 The Relationship Between the Width of the Pile Embedded in Clayey Soil and the Modulus of Subgrade Reaction for all Values of q .

C.5 The Relation Between q and the Modulus of Subgrade Reaction for Different Pile's Widths Embedded in Clayey Soil

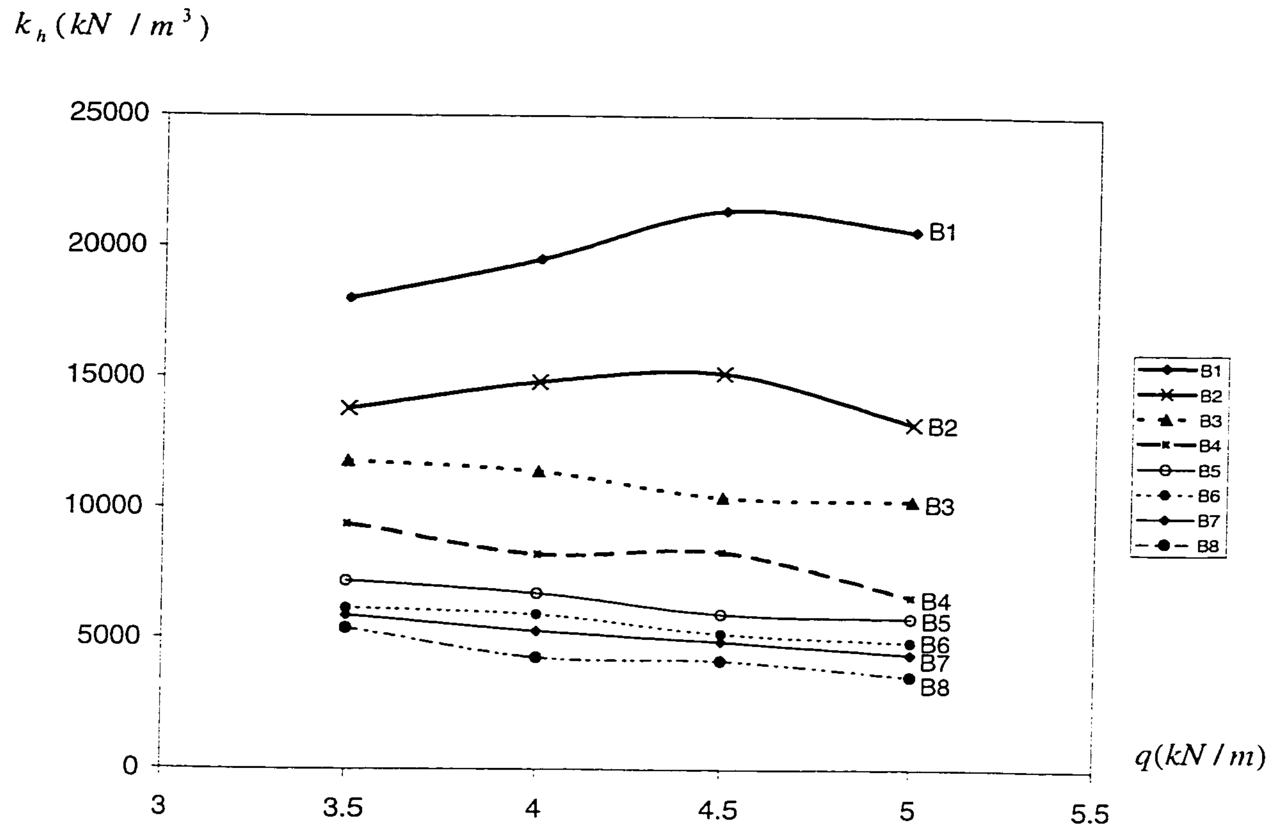


Figure C.11 The Relation Between q and the Modulus of Subgrade Reaction for Different Pile's Widths Embedded in Clayey Soil.

C.6 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil

C.6.1 Width of Pile = 6.35 mm

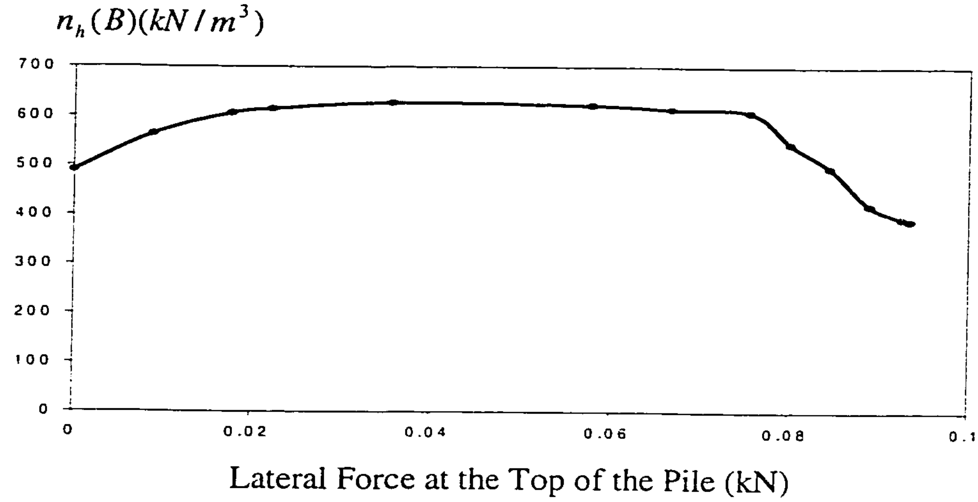


Figure C.12 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6.2 Width of Pile = 12.7 mm

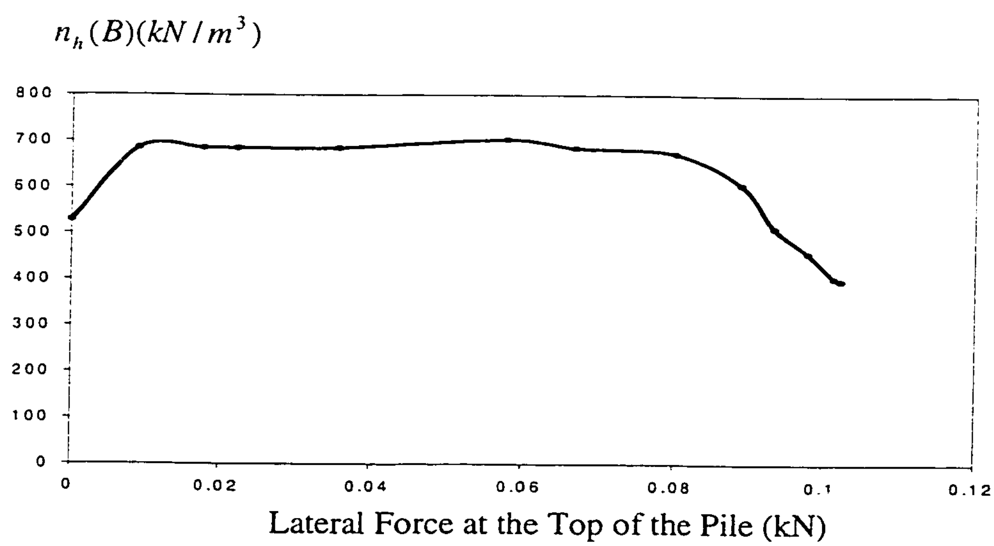


Figure C.13 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil

C.6.3 Width of Pile = 19.05 mm

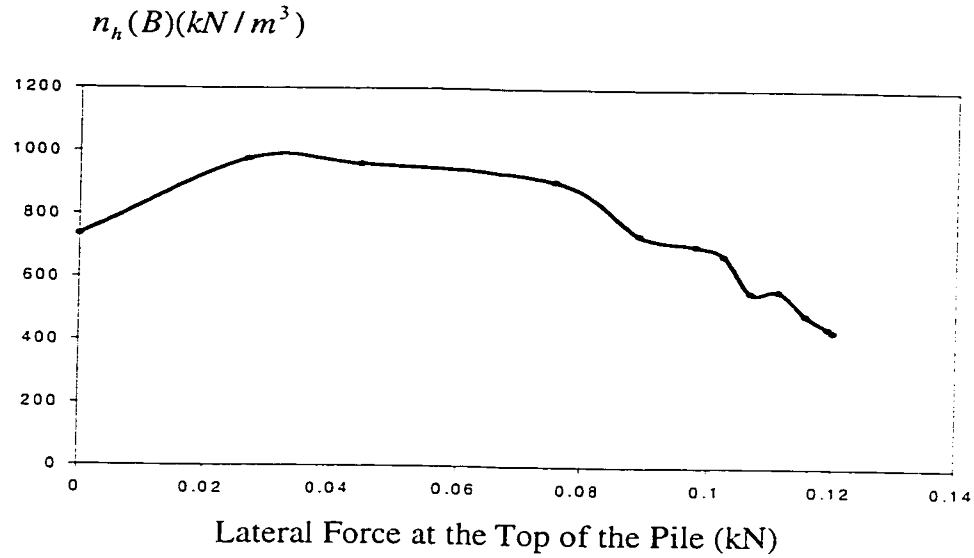


Figure C.14 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6.4 Width of Pile = 25.4 mm

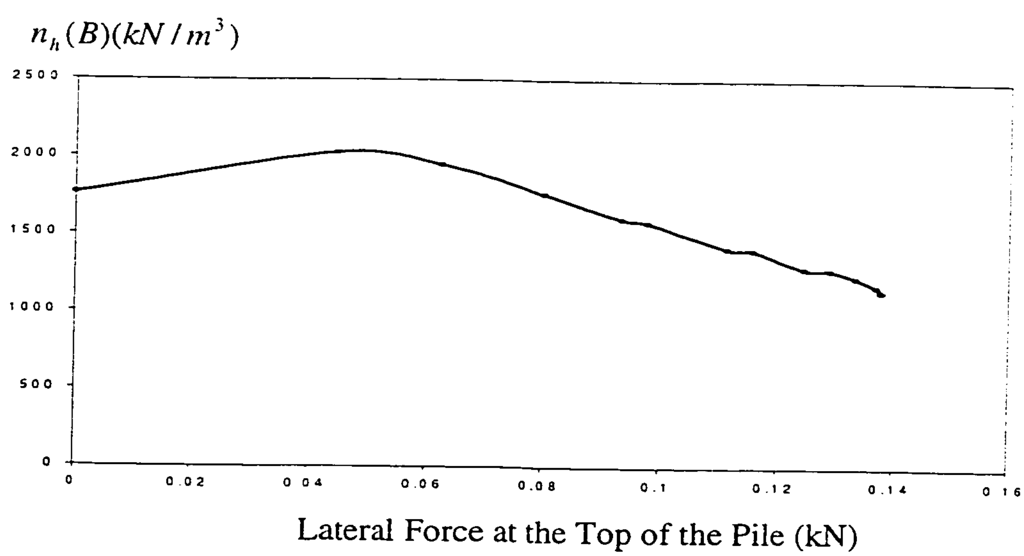
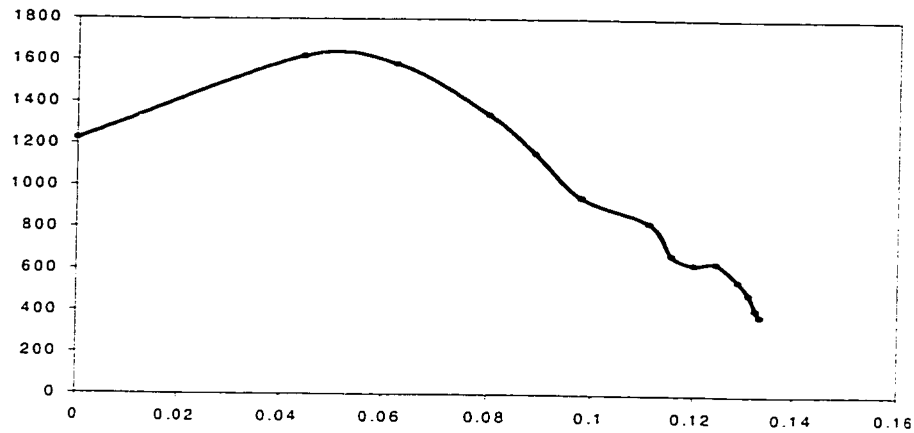


Figure C.15 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil

C.6.5 Width of Pile = 31.75 mm

$n_h(B)(kN/m^3)$

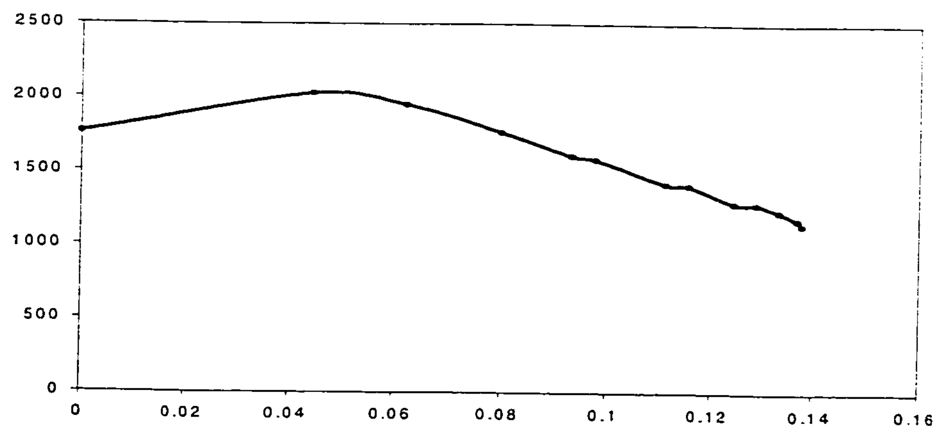


Lateral Force at the Top of the Pile (kN)

Figure C.16 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6.6 Width of Pile = 38.1 mm

$n_h(B)(kN/m^3)$



Lateral Force at the Top of the Pile (kN)

Figure C.17 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil

C.6.7 Width of Pile = 44.45 mm

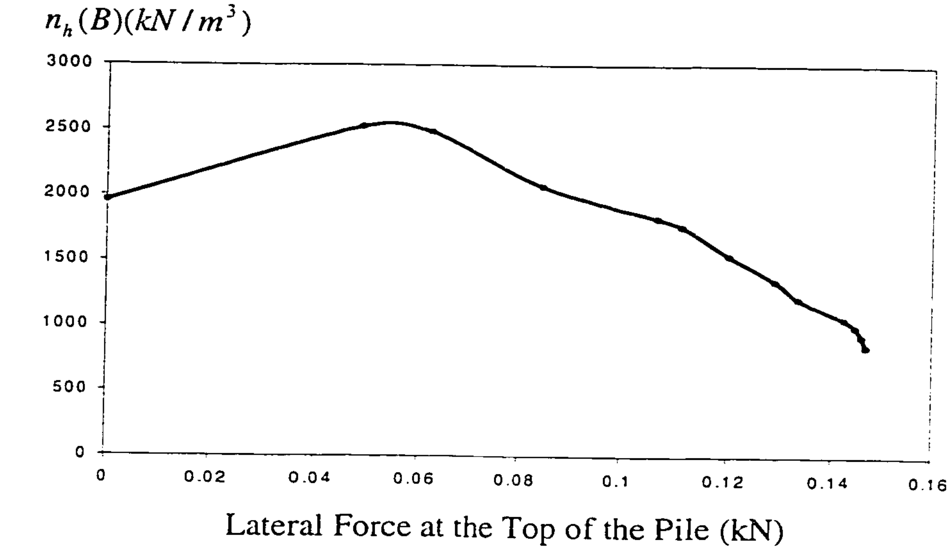


Figure C.18 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.6.8 Width of Pile = 50.8 mm

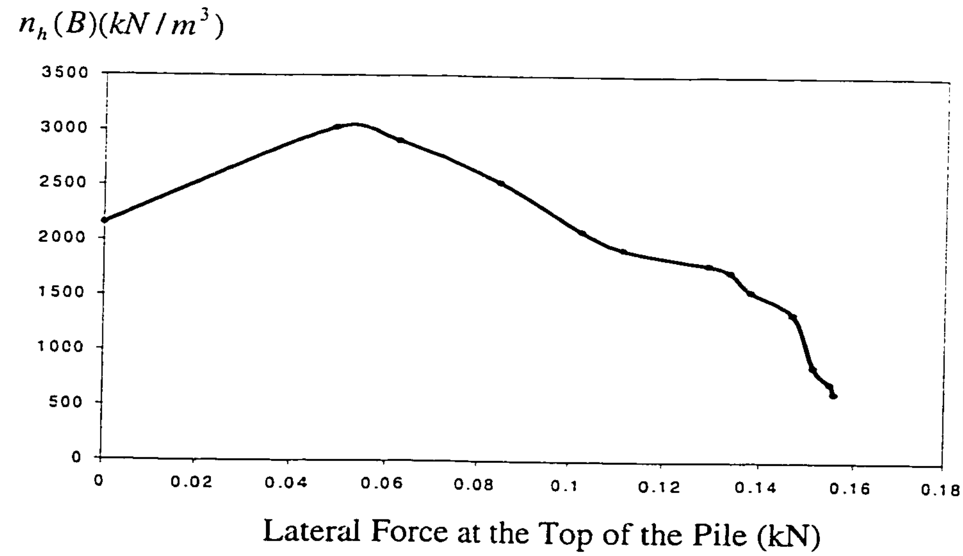
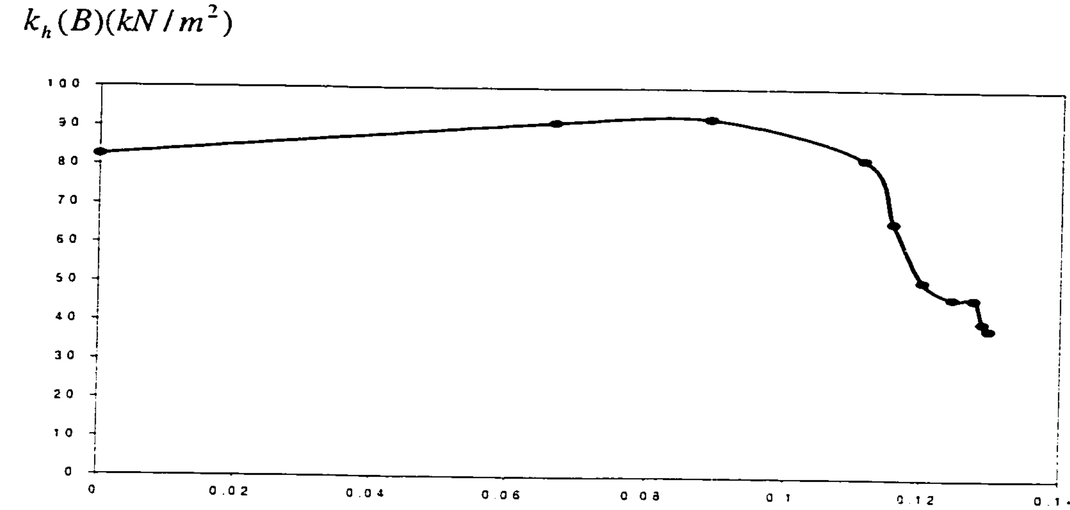


Figure C.19 The Relation Between Lateral Forces and the Constant of Horizontal of Subgrade Reaction for Pile Embedded in Sandy Soil.

C.7 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil

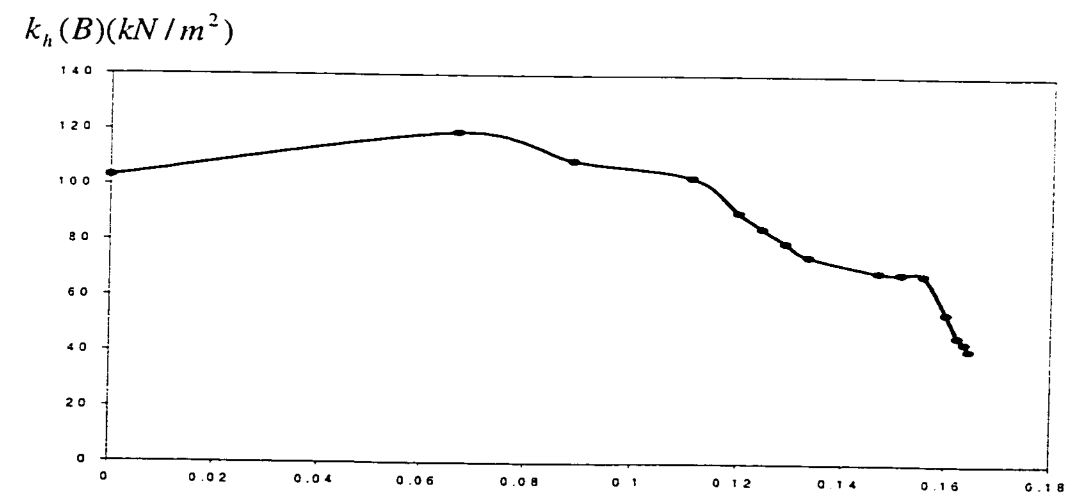
C.7.1 Width of Pile = 6.35 mm



Lateral Force at the Top of the Pile (kN)

Figure C.20 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7.2 Width of Pile = 12.7 mm



Lateral Force at the Top of the Pile (kN)

Figure C.21 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil

C.7.3 Width of Pile = 19.05 mm

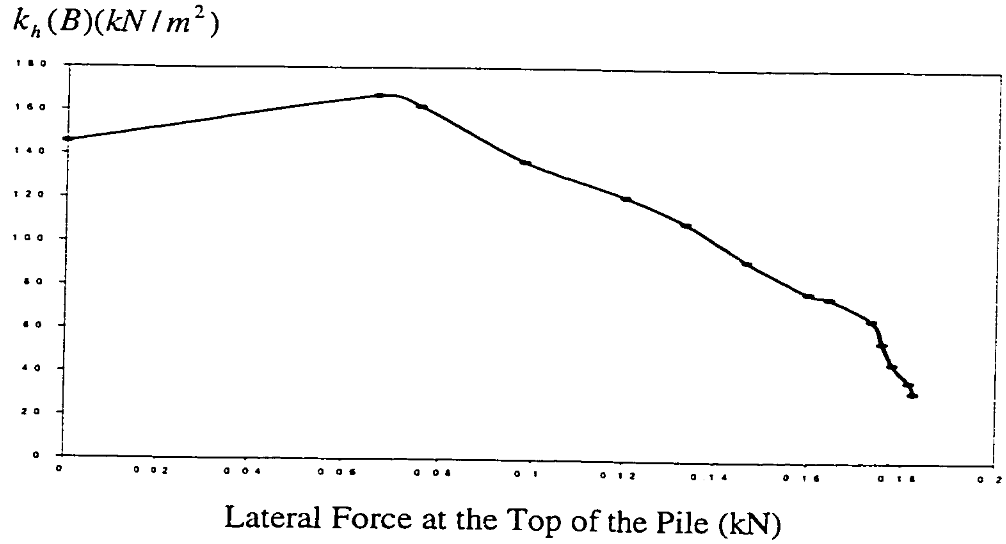


Figure C.22 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7.4 Width of Pile = 25.4 mm

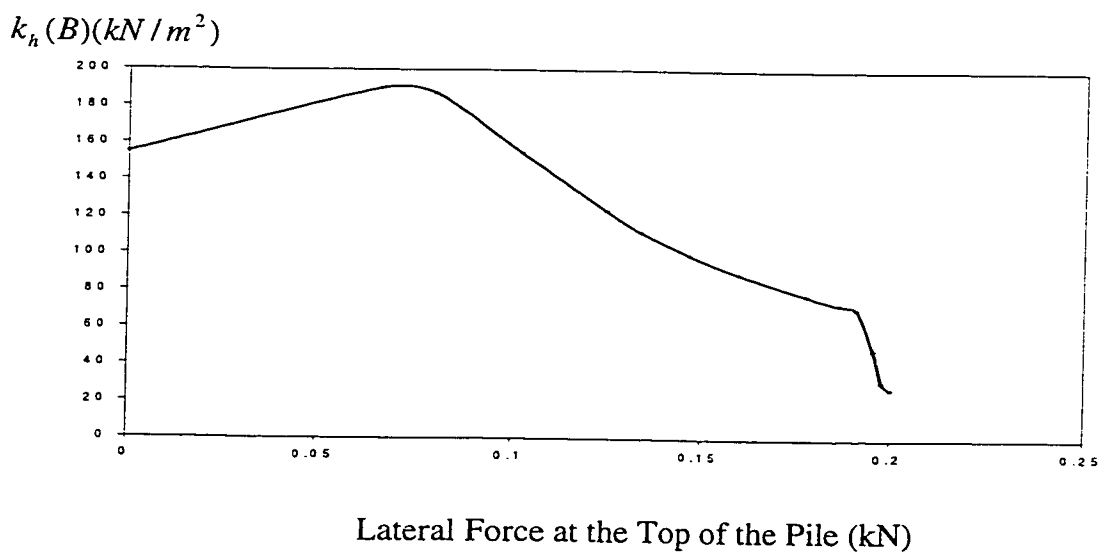


Figure C.23 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil

C.7.5 Width of Pile = 31.75 mm

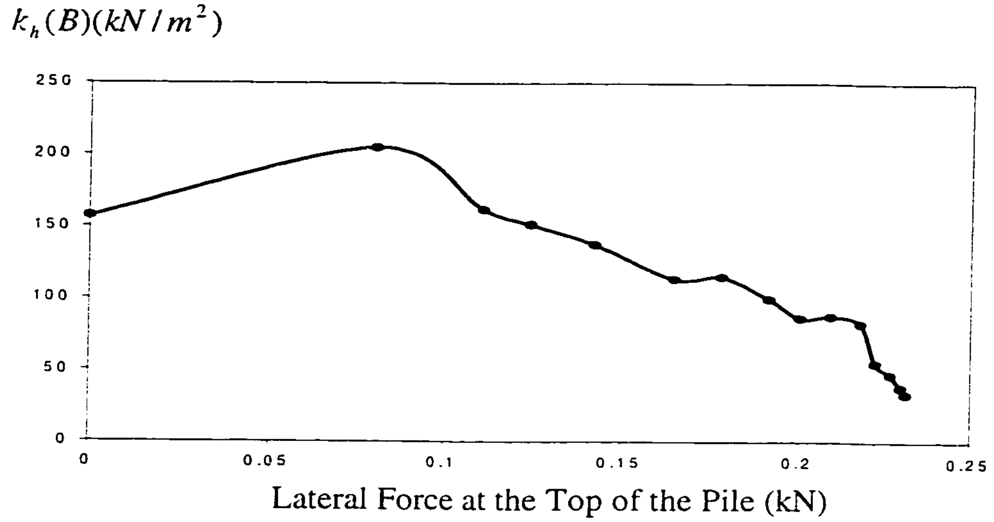


Figure C.24 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7.6 Width of Pile = 38.1 mm

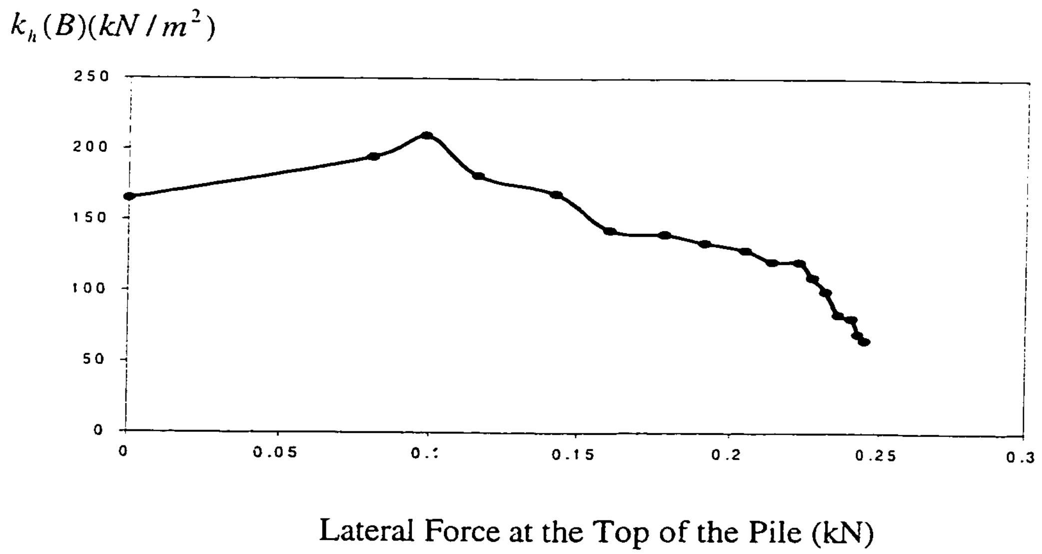


Figure C.25 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil

C.7.7 Width of Pile = 44.45 mm

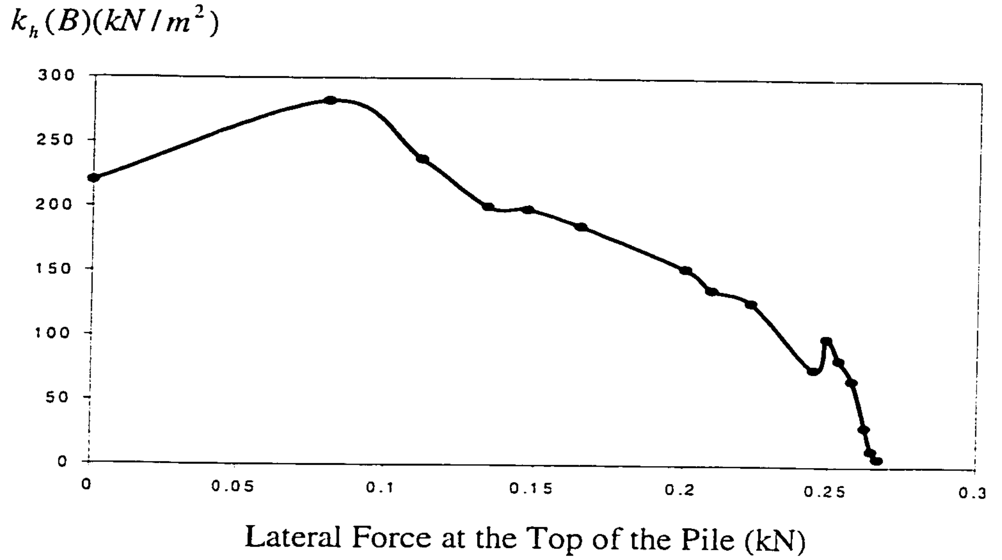


Figure C.26 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

C.7.8 Width of Pile = 50.8 mm

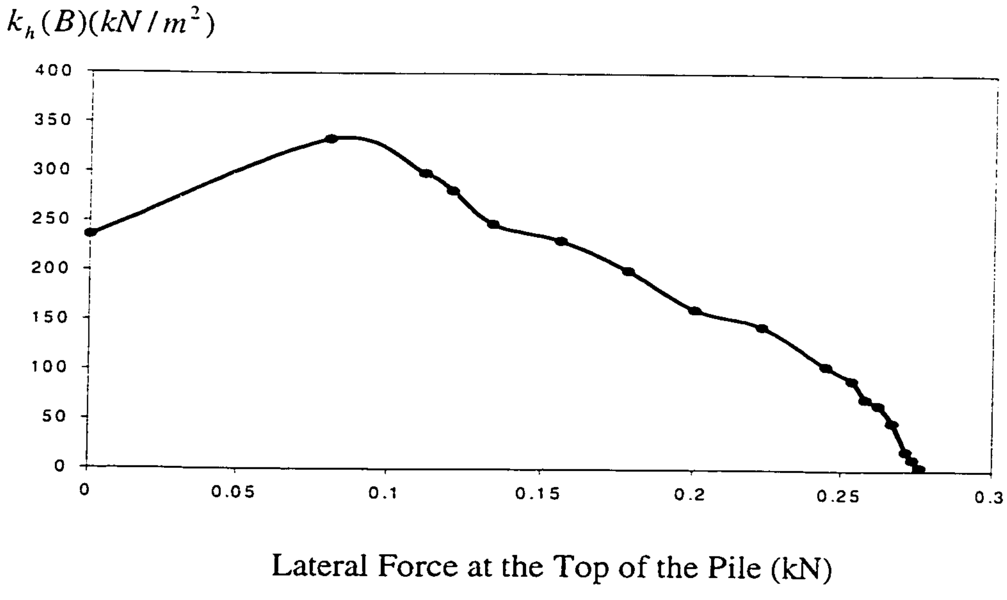


Figure C.27 The Relation Between Lateral Forces and the Modulus of Subgrade Reaction for Pile Embedded in Clayey Soil.

Appendix D

Measurements of Engineering Properties of Soils

D.1 Determination of Water Content for the Investigated Soils

The following tables show the values of water content for different soils.

D.1.1 Sandy Soil

Table D.1 Evaluation of the Water Content of Sandy Soil with 1.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	22	4	900	3
Wt.sample+tare wet (g)	169.0	173.0	157.5	166.5
Wt.sample+tare dry (g)	166.0	169.5	154.0	163.5
Wt. of water (g)	3.0	3.5	3.5	3.0
Wt. of tare (g)	27.5	36.5	28.0	36.5
Wt. of dry soil (g)	138.5	133.0	126.0	127.0
Water content %	2.16%	2.63%	2.77%	2.36%

D.1.2.1 Soil Type 1(with 1.5 % Lime)

Table D.2 Evaluation of the Water Content of Sample Type 1 with 1.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	22	4	900	2
Wt.sample+tare wet (g)	215.5	187.5	195.5	187.5
Wt.sample+tare dry (g)	174.5	147.0	147.0	131.0
Wt. of water (g)	41	40.5	48.5	56.5
Wt. of tare (g)	27.5	36.5	28.0	20.0
Wt. of dry soil (g)	147	110.5	119.0	111.0
Water content %	27.9%	36.66%	40.7%	50.9%

D.1.2.2 Soil Type 1(with 2.0 % Lime)

Table D.3 Evaluation of the Water Content of Sample Type 1 with 2.0% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	22	4	900	3
Wt.sample+tare wet (g)	240.0	187.5	178.5	188.0
Wt.sample+tare dry (g)	199.0	148.5	135.0	143.5
Wt. of water (g)	41.0	39.0	43.5	44.5
Wt. of tare (g)	27.5	36.5	28.0	36.5
Wt. of dry soil (g)	171.5	112	107.0	107.0
Water content %	23.9%	34.8%	40.65%	41.6%

D.1.2.3 Soil Type 1(with 2.5 % Lime)

Table D.4 Evaluation of the Water Content of Sample Type 1 with 2.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	22	4	900	2
Wt.sample+tare wet (g)	215.5	222.0	165.5	183.5
Wt.sample+tare dry (g)	175.5	176.5	125.5	126.5
Wt. of water (g)	40.0	45.5	440.0	57.0
Wt. of tare (g)	27.5	36.5	28.0	20.0
Wt. of dry soil (g)	135.5	131.0	97.5	106.5
Water content %	29.5%	34.73%	41.02%	53.5%

D.1.2.4 Soil Type 2(with 1.5 % Lime)

Table D.5 Evaluation of the Water Content of Sample Type 2 with 1.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	2	13	12	14
Wt.sample+tare wet (g)	220.5	207.5	133.5	150.0
Wt.sample+tare dry (g)	189.0	157.5	101.0	106.0
Wt. of water (g)	31.5	50.0	32.5	44.0
Wt. of tare (g)	20.0	28.8	20.0	21.0
Wt. of dry soil (g)	169.0	129.5	81.0	85.0
Water content %	18.6%	38.6%	40.1%	51.7%

D.1.2.5 Soil Type 2(with 2.0 % Lime)

Table D.6 Evaluation of the Water Content of Sample Type 2 with 2.0% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	900	13	12	34
Wt.sample+tare wet (g)	220.0	217.5	195.1	154.0
Wt.sample+tare dry (g)	179.0	171.0	146.0	112.5
Wt. of water (g)	41.0	46.5	49.1	41.5
Wt. of tare (g)	28.0	28.0	20.0	22.0
Wt. of dry soil (g)	151.0	143.0	126	900.5
Water content %	27.15%	32.5%	39.0%	45.8%

D.1.2.6 Soil Type 2(with 2.0 % lime)

Table D.7 Evaluation of the Water Content of Sample Type 2 with 2.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	900	13	12	34
Wt.sample+tare wet (g)	210.0	204.5	183.0	180.5
Wt.sample+tare dry (g)	172.0	158.0	134.5	130.5
Wt. of water (g)	38.0	46.5	48.5	50.0
Wt. of tare (g)	28.0	28.0	20.0	22.0
Wt. of dry soil (g)	144.0	130.0	114.5	108.5
Water content %	26.4%	35.7%	42.3%	46.1%

D.1.2.7 Soil Type 3(with 1.5 % Lime)

Table D.8 Evaluation of the Water Content of Sample Type 3 with 1.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	4	900	3	2
Wt.sample+tare wet (g)	205.0	199.0	187.5	160.5
Wt.sample+tare dry (g)	161.0	152.5	142.5	113.5
Wt. of water (g)	44.0	46.5	45.0	47.5
Wt. of tare (g)	36.5	28.0	36.5	20.0
Wt. of dry soil (g)	124.5	124.5	106.0	93.5
Water content %	35.34%	37.4%	42.45%	50.26%

D.1.2.8 Soil Type 3(with 2.0 % Lime)

Table D.9 Evaluation of the Water Content of Sample Type 3 with 2.0% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	4	900	3	2
Wt.sample+tare wet (g)	221.5	201.0	238.5	193.5
Wt.sample+tare dry (g)	174.5	153.5	180.5	136.5
Wt. of water (g)	47.5	47.5	58.0	57.0
Wt. of tare (g)	36.5	28.0	36.5	20.0
Wt. of dry soil (g)	138.0	125.5	144.0	116.5
Water content %	34.1%	37.8%	40.3%	48.9%

D.1.2.9 Soil Type 3(with 2.5 % lime)

Table D.10 Evaluation of the Water Content of Sample Type 3 with 2.5% Lime

Sample number	1	2	3	4
Type of test	Oven dry method			
Container number	4	900	3	2
Wt.sample+tare wet (g)	207.5	184.5	156.5	180.0
Wt.sample+tare dry (g)	163.0	142.0	121.5	127.5
Wt. of water (g)	44.5	42.5	35.0	52.5
Wt. of tare (g)	36.5	28.0	36.5	20.0
Wt. of dry soil (g)	126.5	114	85.0	107.5
Water content %	35.2%	37.28%	41.17%	48.8%

D.2 Determination of Liquid Limit for the Investigated Soils

D.2.1 Soil Type 1

Table D.11 Evaluation of Liquid Limit of Sample Type 1

Can no.	2	5	1	7
Wt.of wet soil+can (g)	174.5	162.4	189.1	246.5
Wt.of dry soil+can (g)	124.7	121	149.5	211.5
Wt.of can (g)	28	28	36.5	36.5
Wt.of dry soil (g)	96.7	93	113	175
Wt.of moisture (g)	49.8	41.4	39.6	35
Water content,w%	51.5	44.5	35	20
No.of blows,N	13	20	34	60

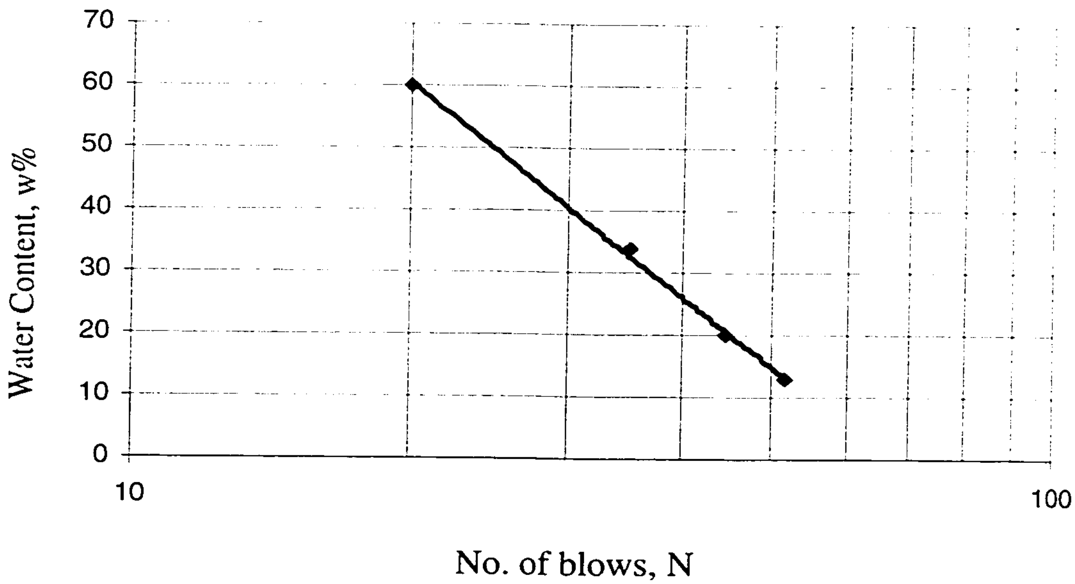


Figure D.1 Determination of Liquid Limit for Soil Type 1.

- **Liquid Limit = 47 %**

D.2.2 Soil Type 2

Table D.12 Evaluation of Liquid Limit of Sample Type 2

Can no.	2	5	1	7
Wt.of wet soil+can (g)	190.2	179.9	160.62	158.42
Wt.of dry soil+can (g)	137	136.5	127.1	138.1
Wt.of can (g)	28	28	36.5	36.5
Wt.of dry soil (g)	109	108.5	90.6	101.6
Wt.of moisture(g)	53.2	43.4	33.52	20.32
Water content,w%	48.8	40	37	20
No.of blows,N	12	20	27	60

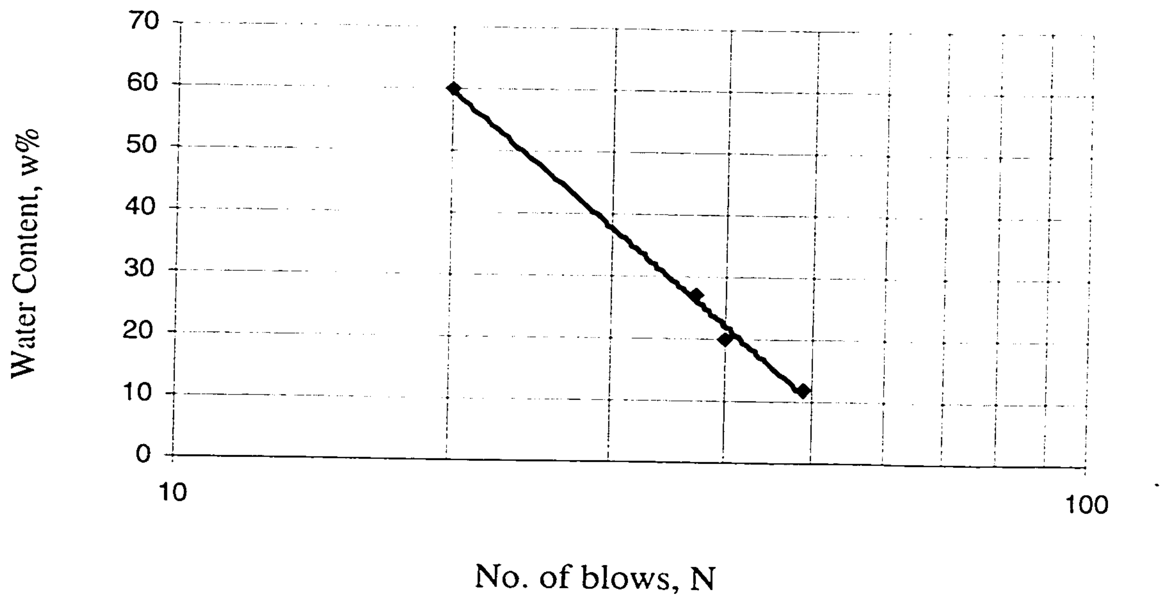


Figure D.2 Determination of Liquid Limit for Soil Type 2.

- **Liquid Limit = 49 %**

D.2.3 Soil Type 3

Table D.13 Evaluation of Liquid Limit of Sample Type 3

Can no.	1	2	3	4
Wt.of wet soil+can (g)	131.86	148.58	143.28	156.25
Wt.of dry soil+can (g)	98.26	111.16	115.56	133.73
Wt.of can (g)	36.5	28	27	27
Wt.of dry soil (g)	61.76	83.16	88.56	106.73
Wt.of moisture (g)	33.6	37.42	27.72	22.52
Water content,w%	54.4	45	31.3	21.1
No.of blows,N	12	20	33	63

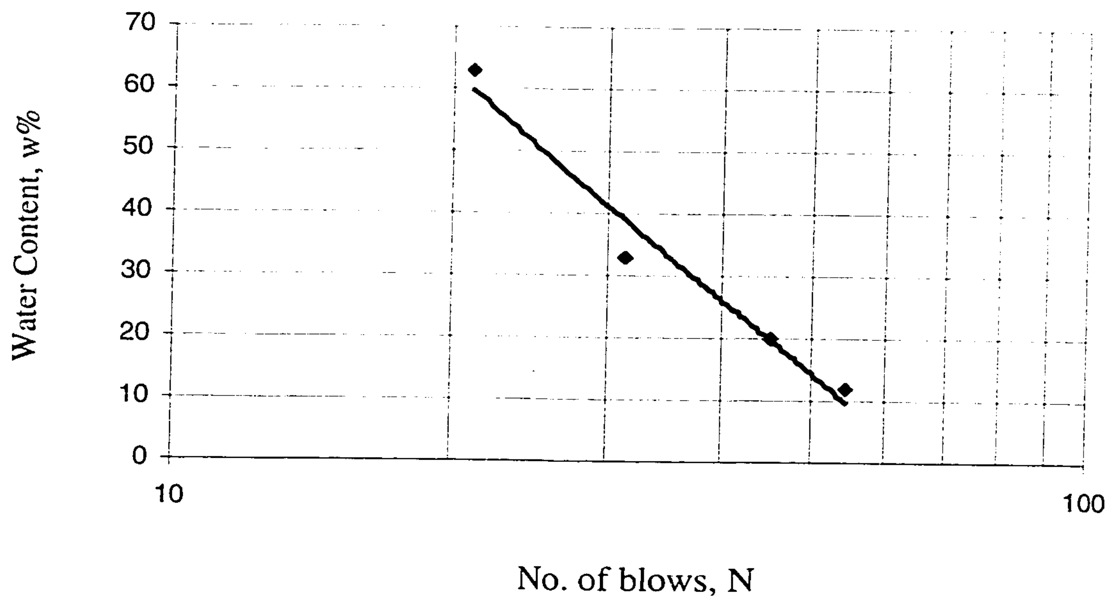


Figure D.3 Determination of Liquid Limit for Soil Type 3.

- **Liquid Limit = 51 %**

D.3 Determination of Plastic Limit for the Investigated Soils

D.3.1.Soil Type 1

Table D.14 Evaluation of Plastic Limit of Sample Type 1

Can no.	1	2	3
Wt.of wet soil+can (g)	10.31	11.5	12.9
Wt.of dry soil+can (g)	10.02	11.18	12.54
Wt.of can (g)	8.2	9.22	10.22
Wt.of dry soil (g)	1.82	1.96	2.32
Wt.of moisture (g)	0.29	0.32	0.36
Water content,w%	15.93	16.33	15.52

- **Plastic Limit. = 15.9 %**

D.3.2 Soil Type 2

Table D.15 Evaluation of Plastic Limit of Sample Type 2

Can no.	4	5	6
Wt.of wet soil+can (g)	9.57	10.72	9.77
Wt.of dry soil+can (g)	9.33	10.43	9.46
Wt.of can (g)	7.9	8.72	7.66
Wt.of dry soil (g)	1.43	1.71	1.8
Wt.of moisture (g)	0.24	0.29	0.31
Water content,w%	16.78	16.96	17.22

- **Plastic Limit. = 17.0 %**

D.3.3 Soil Type 3

Table D.16 Evaluation of Plastic Limit of Sample Type 3

Can no.	1	2	3
Wt.of wet soil+can (g)	10.89	12.13	12.98
Wt.of dry soil+can (g)	10.5	11.7	12.56
Wt.of can (g)	8.2	9.22	10.22
Wt.of dry soil (g)	2.3	2.48	2.34
Wt.of moisture (g)	0.39	0.43	0.42
Water content,w%	16.96	17.34	17.95

- **Plastic Limit. = 17.4 %**

D.4 Determination of Grain Size Distribution for the Investigated Soils

D.4.1Soil Type 1

Table D.17 Determination % Passing for Grain Size Distribution of Sample Type 1

Sieve no.	Diam. (mm)	Wt.retained (g)	% retained	% passing
4	4.76	-	0	100
10	2	-	0	100
20	0.84	8.85	1.77	98.23
40	0.42	71.2	14.24	83.99
60	0.25	198.35	39.67	44.32
100	0.149	111.5	22.3	22.02
200	0.074	75.5	15.1	6.92
Pan		32.2		
		497.6		

$$\% \text{ Losses} = (1 - \frac{497.6}{500}) * 100 = 0.48\%$$

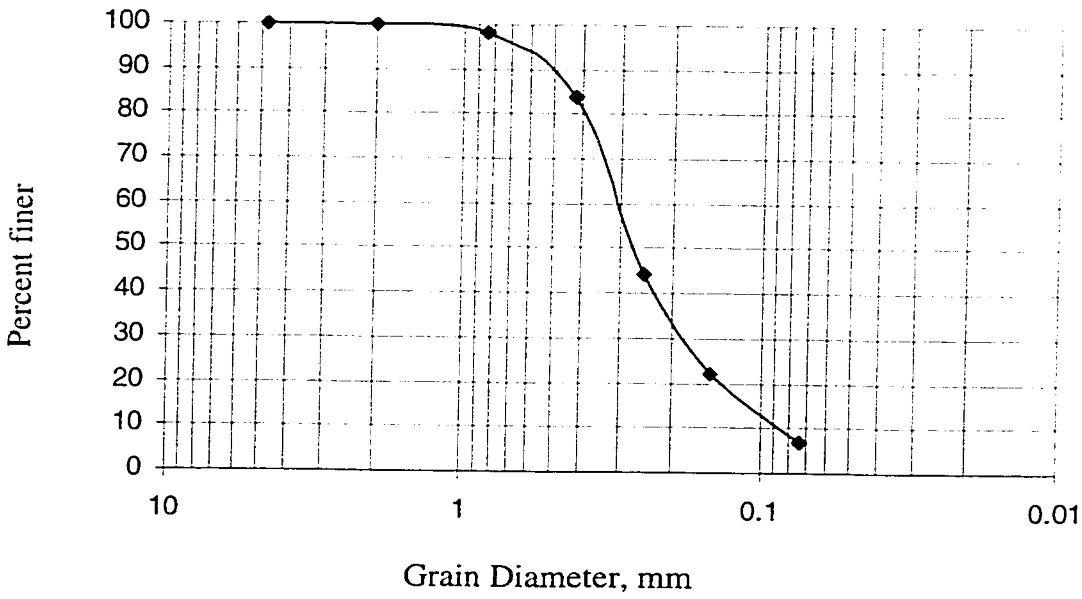


Figure D.4 Typical Grain Size Curve for Soil Type 1.

D.4.2 Soil Type 2

Table D.18 Determination % Passing for Grain Size Distribution of Sample Type 2

Sieve no.	Diam. (mm)	Wt.retained (g)	% retained	% passing
4	4.76	-	0	100
10	2	-	0	100
20	0.84	12.85	2.57	97.43
40	0.42	62.5	12.5	84.93
60	0.25	206.65	41.33	43.6
100	0.149	131.5	26.3	17.3
200	0.074	44.5	8.9	8.4
Pan		40.5		
		498.5		

$$\% \text{ Losses} = (1 - \frac{498.5}{500}) * 100 = 0.3\%$$

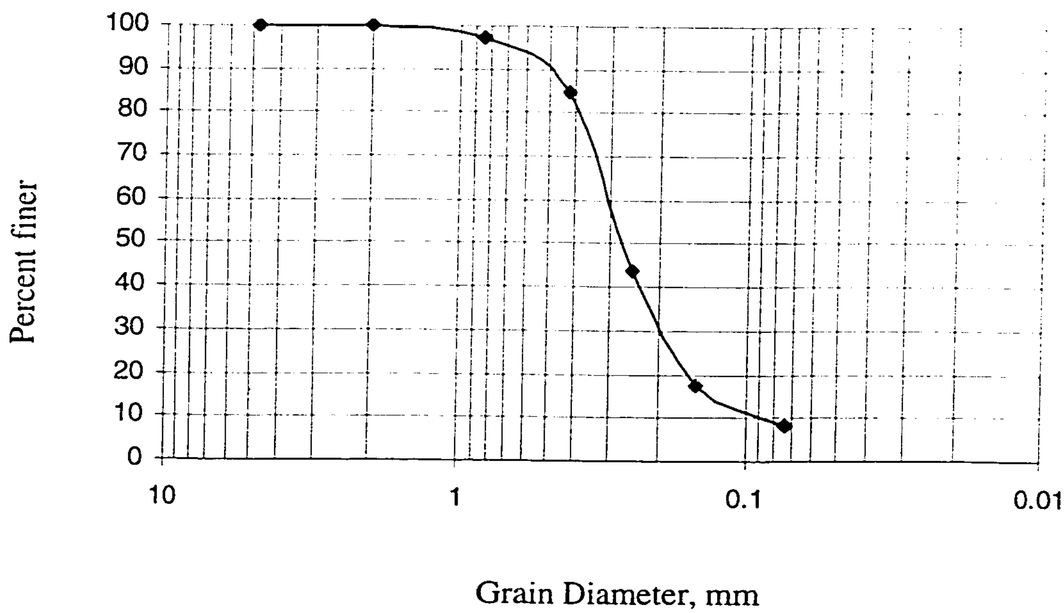


Figure D.5 Typical Grain Size Curve for Soil Type 2.

D.4.3 Soil Type 3

Table D.19 Determination % Passing for Grain Size Distribution of Sample Type 3

Sieve no.	Diam. (mm)	Wt.retained (g)	% retained	% passing
4	4.76	-	0	100
10	2	-	0	100
20	0.84	11.9	2.38	97.62
40	0.42	106.7	21.34	76.28
60	0.25	97.8	19.56	56.72
100	0.149	82.1	16.42	40.3
200	0.074	118	23.6	16.7
Pan		81.6		
		498.1		

$$\% \text{ Losses} = (1 - \frac{498.1}{500}) * 100 = 0.38\%$$

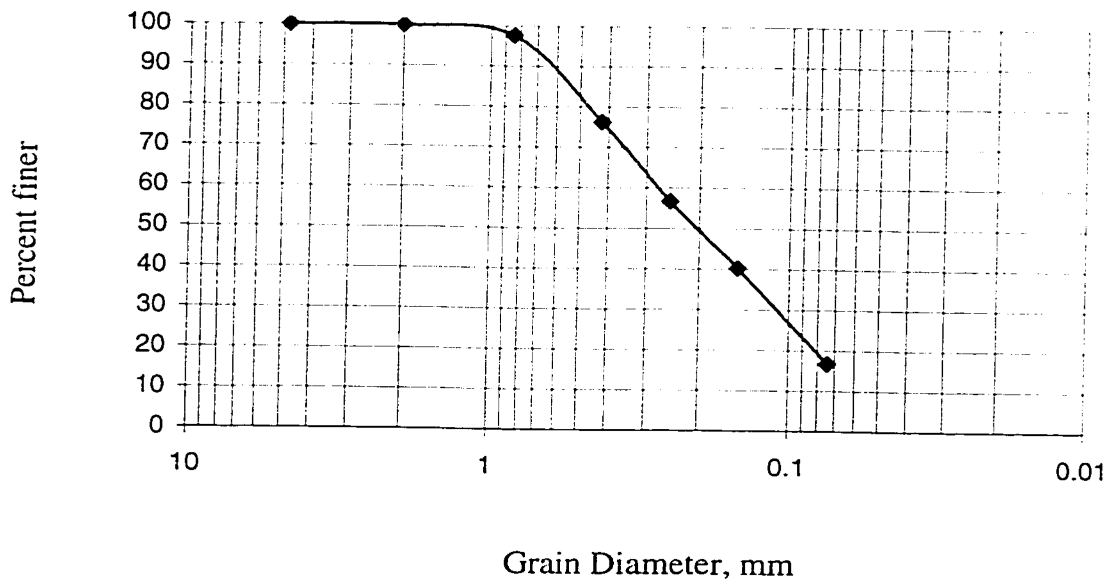


Figure D.6 Typical Grain Size Curve for Soil Type 3.

D.5 Determination of the Unconfined Compressive Strength for the Investigated Soils

D.5.1 Soil Type 1

Table D.20 Evaluation the Unconfined Compressive Strength of Sample Type 1

Deformation dial reading	Load dial (units)	Sample deformation ΔL(in)	Unit strain $\Delta L/L_o$	1-ε	Corrected Area(A') (in^2)	Total load on sample (lb)	Sample unit ksf
0	0	0	0	1	2.138	0	0
10	0.6	0.01	0.0027	0.9973	2.1437	0.7662	0.0515
20	1.3	0.02	0.0054	0.9946	2.1496	1.6601	0.111
30	2.1	0.03	0.0081	0.9919	2.155	2.6817	0.179
40	2.7	0.04	0.011	0.989	2.162	3.4479	0.2296
50	3.6	0.05	0.0134	0.9866	2.167	4.5972	0.305
60	4.4	0.06	0.016	0.984	2.173	5.6188	0.3723
70	4.7	0.07	0.0188	0.9812	2.179	6	0.3966
80	5.1	0.08	0.0215	0.9785	2.185	6.5127	0.4292
90	5.4	0.09	0.0242	0.9758	2.191	6.8958	0.453
100	5.9	0.1	0.02688	0.97312	2.197	7.5343	0.494
110	6	0.11	0.0295	0.9704	2.203	7.662	0.5
120	6	0.12	0.03225	0.96775	2.209	7.662	0.499

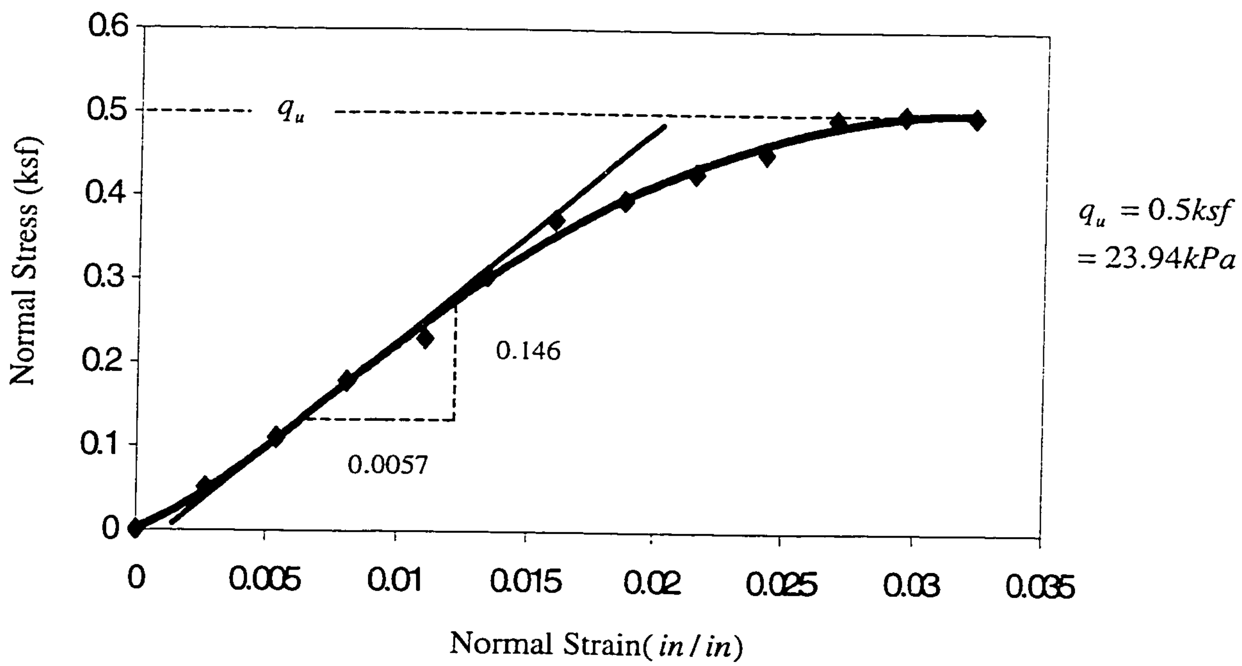
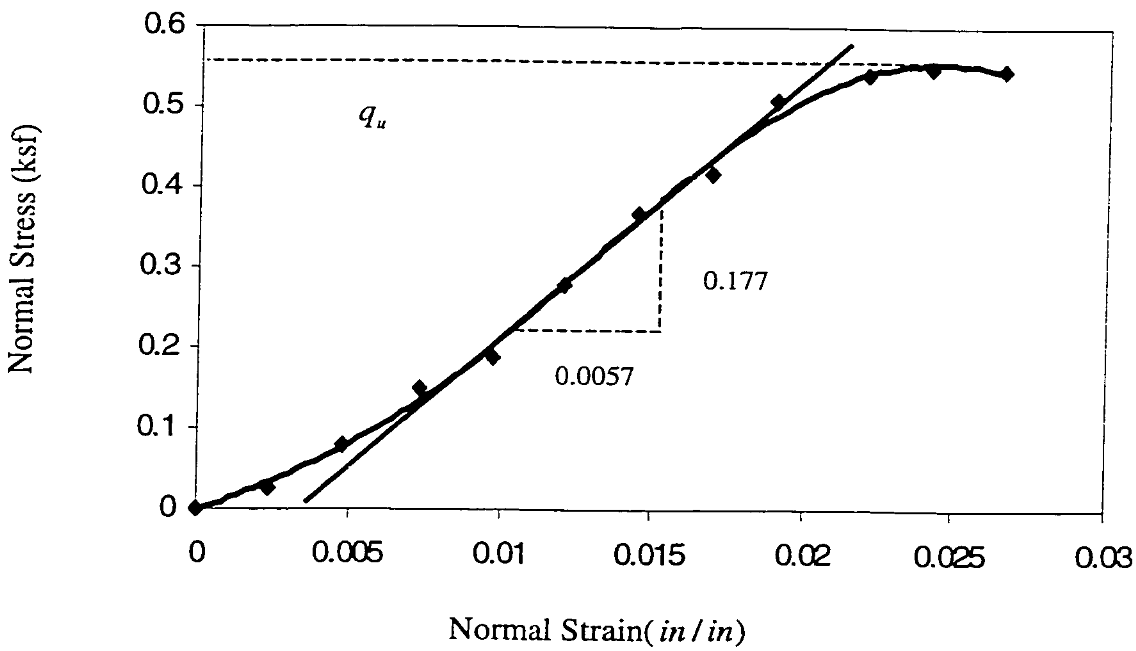


Figure D.7 Typical Plot of Unconfined Compression Test Data for Soil Type 2.

D.5.2 Soil Type 2

Table D.21 Evaluation the Unconfined Compressive Strength of Sample Type 2

Deformation dial reading	Load dial (units)	Sample deformation ΔL(in)	Unit strain $\Delta L/L_O$	1-ε	Corrected Area(A') (in^2)	Total load on sample (lb)	Sample unit ksf
0	0	0	0	1	2.8055	0	0
10	0.4	0.01	0.0024	0.9976	2.8122	0.511	0.026
20	1.2	0.02	0.0048	0.9952	2.819	1.532	0.078
30	2.3	0.03	0.0073	0.9927	2.826	2.937	0.15
40	2.9	0.04	0.0097	0.9903	2.833	3.7	0.188
50	4.3	0.05	0.0121	0.9879	2.839	5.491	0.279
60	5.7	0.06	0.0145	0.9855	2.8467	7.279	0.368
70	6.5	0.07	0.0169	0.9831	2.8537	8.3	0.419
80	7.9	0.08	0.019	0.981	2.859	10.03	0.508
90	8.4	0.09	0.022	0.978	2.868	10.727	0.54
100	8.6	0.1	0.0242	0.9758	2.875	10.9822	0.55
110	8.6	0.11	0.0266	0.9734	2.882	10.9822	0.548



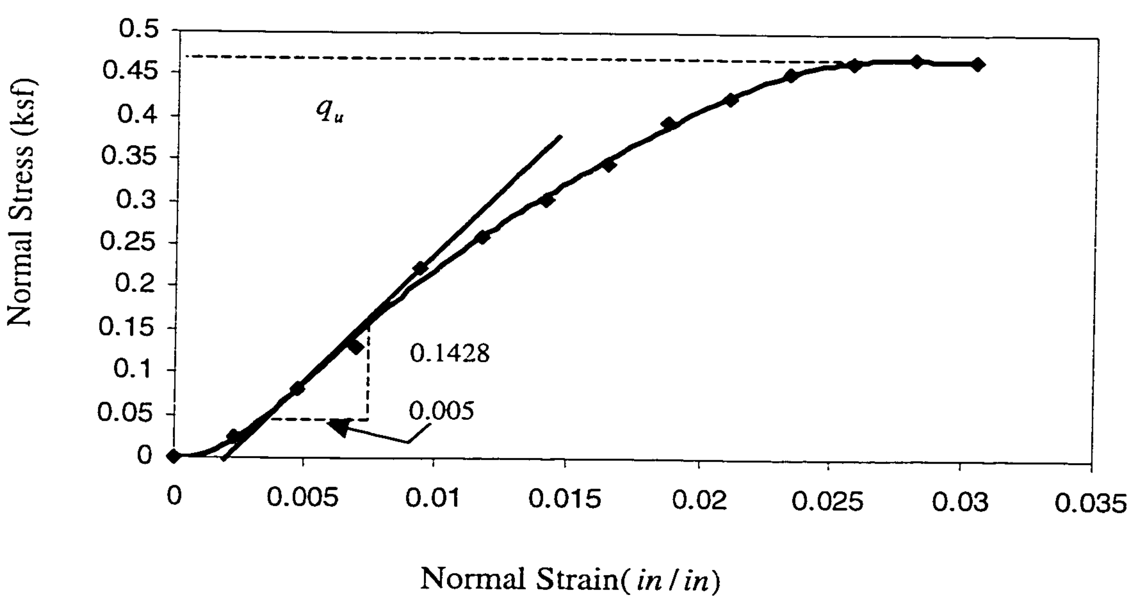
$q_u = 0.55ksf$
$= 26.33kPa$

Figure D.8 Typical Plot of Unconfined Compression Test Data for Soil Type 2.

D.5.3 Soil Type 3

Table D.22 Evaluation the Unconfined Compressive Strength of Sample Type 3

Deformation dial reading	Load dial (units)	Sample deformation ΔL(in)	Unit strain $\Delta L/L_o$	1-ε	Corrected Area(A') (in^2)	Total load on sample (lb)	Sample unit ksf
0	0	0	0	1	2.9255	0	0
10	0.4	0.01	0.0023	0.9977	2.9322	0.511	0.0251
20	1.3	0.02	0.0047	0.9953	2.9393	1.66	0.081
30	2.1	0.03	0.007	0.993	2.9461	2.682	0.131
40	3.6	0.04	0.0094	0.9906	2.9532	4.6	0.224
50	4.2	0.05	0.01171	0.9883	2.9602	5.363	0.261
60	4.9	0.06	0.0141	0.9859	2.9673	6.2573	0.304
70	5.6	0.07	0.0164	0.9836	2.9743	7.151	0.346
80	6.4	0.08	0.0187	0.9813	2.9812	8.173	0.395
90	6.9	0.09	0.0211	0.9789	2.988	8.8113	0.425
100	7.4	0.1	0.0234	0.9766	2.996	9.45	0.454
110	7.6	0.11	0.02576	0.9742	3.0	9.71	0.466
120	7.7	0.12	0.0281	0.9719	3.01	9.833	0.47
130	7.7	0.13	0.03044	0.9696	3.017	9.833	0.469



$q_u = 0.47ksf$
$= 22.5kPa$

Figure D.9 Typical Plot of Unconfined Compression Test Data for Soil Type 3.

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